CEE 551 - Traffic Science

Topic: Traffic Signal Control (4)

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Level of Service Analysis

- Vehicle delays & Level of Service
- Deterministic delay (with uniform arrival)
  - Vehicle delay in queueing diagram
  - Vehicle delay calculation in TS diagram
- Stochastic traffic demand and maximum green
- Stochastic delay

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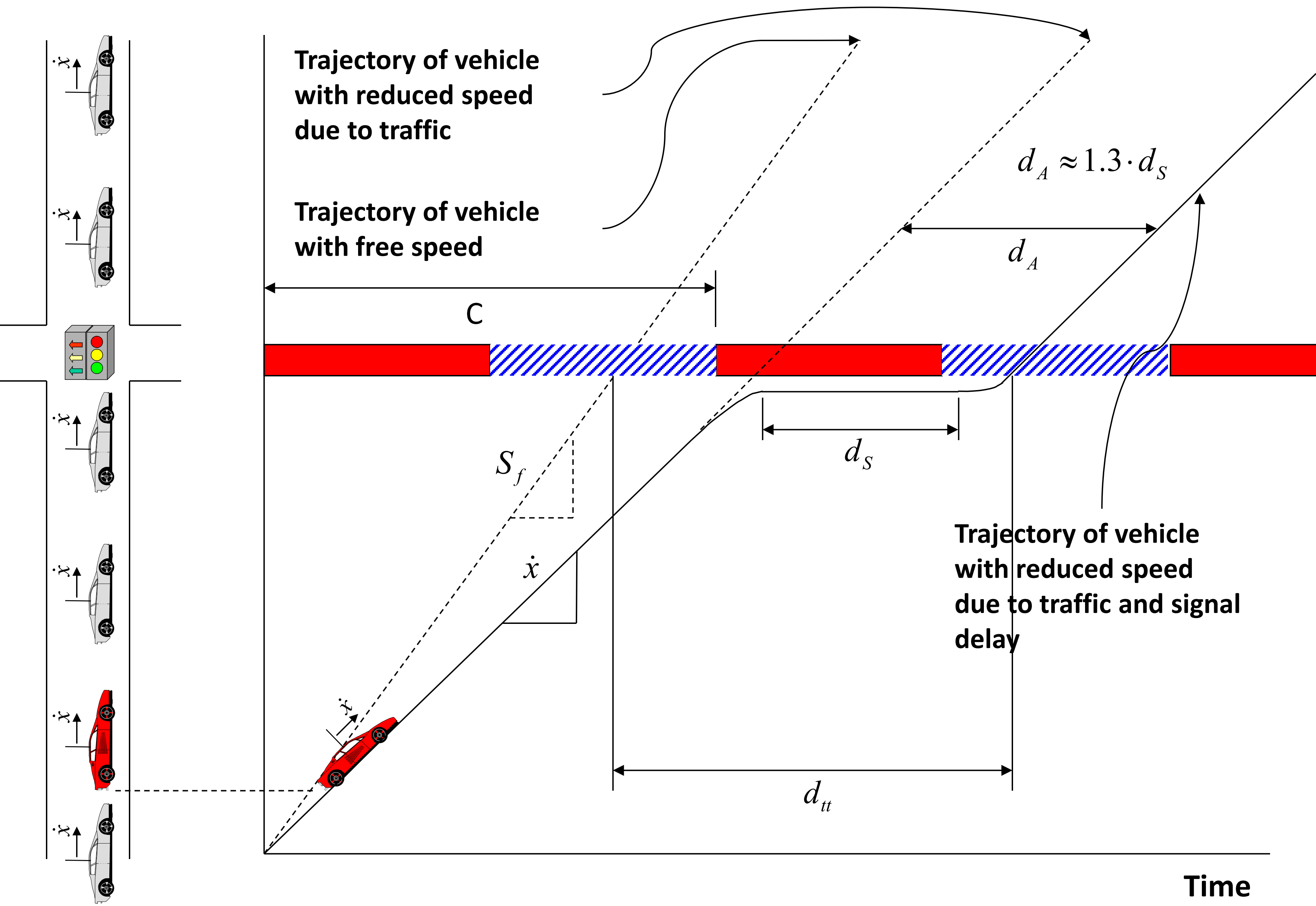
Level of Service Analysis

- Analysis Procedure (assuming signal timing plan has already been determined)
  - Determine the capacities (service) and volumes (arrivals)
  - Calculate delay
  - Determine Level of Service (LOS)
- LOS and Delays

Highway Capacity Manual Level of Service Criteria for Signalized Intersections

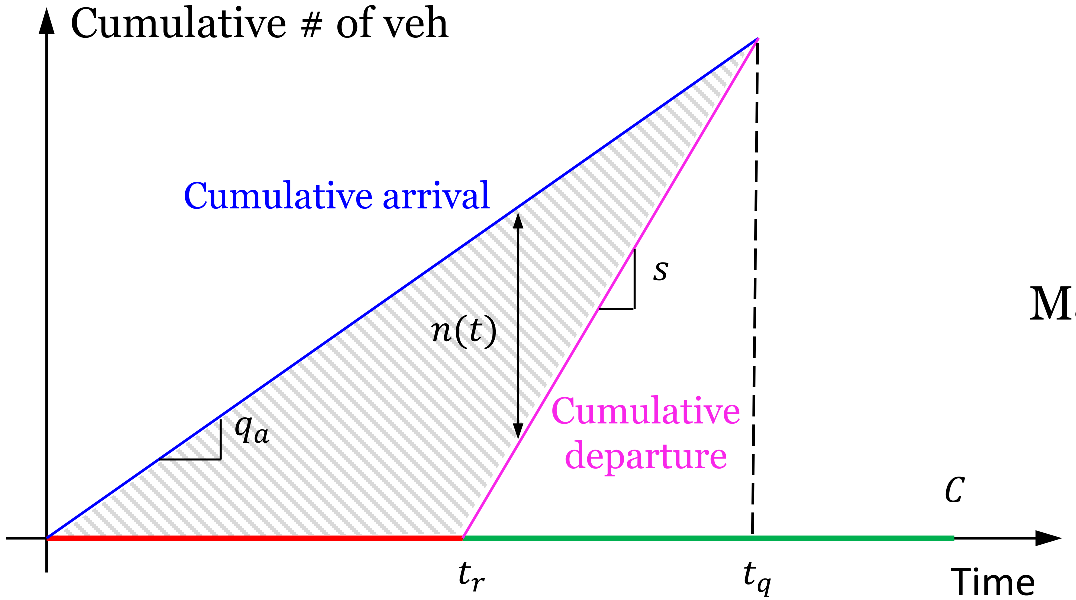
Level of service	Average control delay (seconds/vehicle)	Description
A	⩽10	Free flow
B	>10–20	Stable flow (slight delay)
C	>20–35	Stable flow (acceptable delays)
D	>35–55	Approaching unstable flow (tolerable delay)
E	>55–80	Unstable flow (intolerable delay)
F	>80	Forced flow (congested and queues fail to clear)



Trajectory of vehicle with reduced speed due to traffic
Trajectory of vehicle with free speed
$d_A \approx 1.3 \cdot d_S$
d_A
C
S_f
d_S
$\dot{x}$
Trajectory of vehicle with reduced speed due to traffic and signal delay
d_{tt}
Time

Queueing Diagram with Uniform Arrival

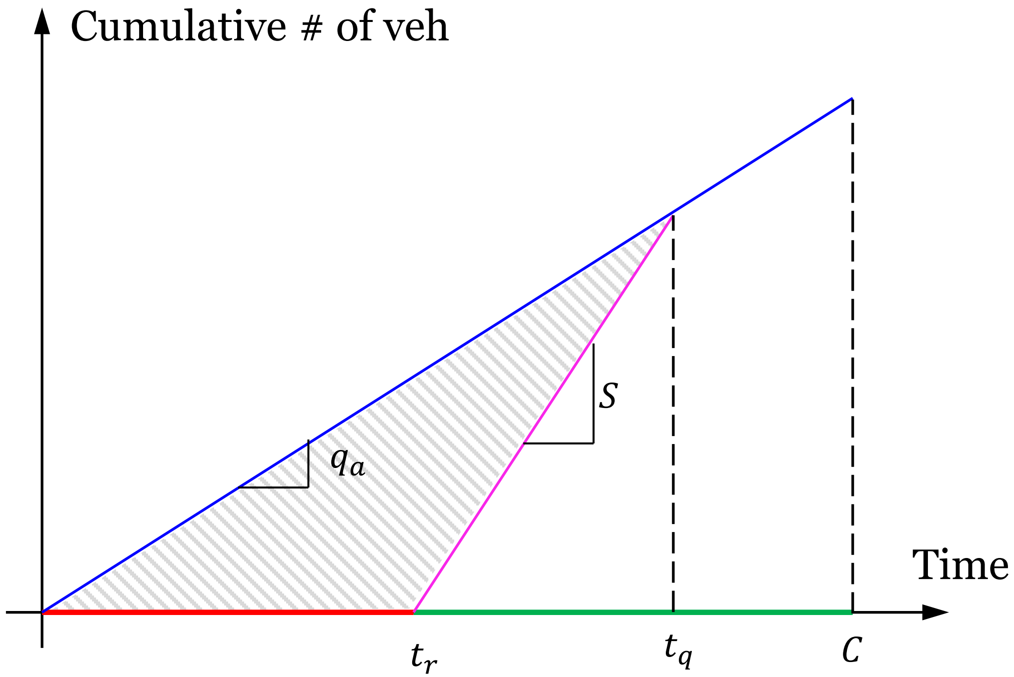
- Queueing diagram



Maximum queue?

  - $n(t)$: number of waiting vehicles at time t
  - Total delay $D = \int n(t)dt$: shadow area, unit: veh*sec

Webster's Delay (Triangular)



- Green time: t_g
- Red time: t_r
- Arrival rate: q_a
- Saturation flow rate: S

$$Y = \frac{q_a}{S}$$

- Queue clearance time t_q

$$S \cdot t_q = q_a(t_r + t_q)$$

$$t_q = \frac{q_a t_r}{S - q_q} = \frac{Y t_r}{1 - Y}$$

- Total delay (shadow area)

$$D = \frac{1}{2} \cdot q_a \cdot \left(t_r + t_q\right) \cdot t_r$$

$$= \frac{1}{2} \cdot \frac{q_a t_r^2}{1 - Y}$$

- Average delay per vehicle

$$d = \frac{D}{q_a C} = \frac{t_r^2}{2C(1 - Y)}$$

Webster's Delay (Triangular)

- Webster's delay (deterministic triangular delay)

$$d = \frac{t_r^2}{2C(1-Y)}$$

Average delay per vehicle

- Change of variables

Green split: $\theta = \frac{t_g}{C}$

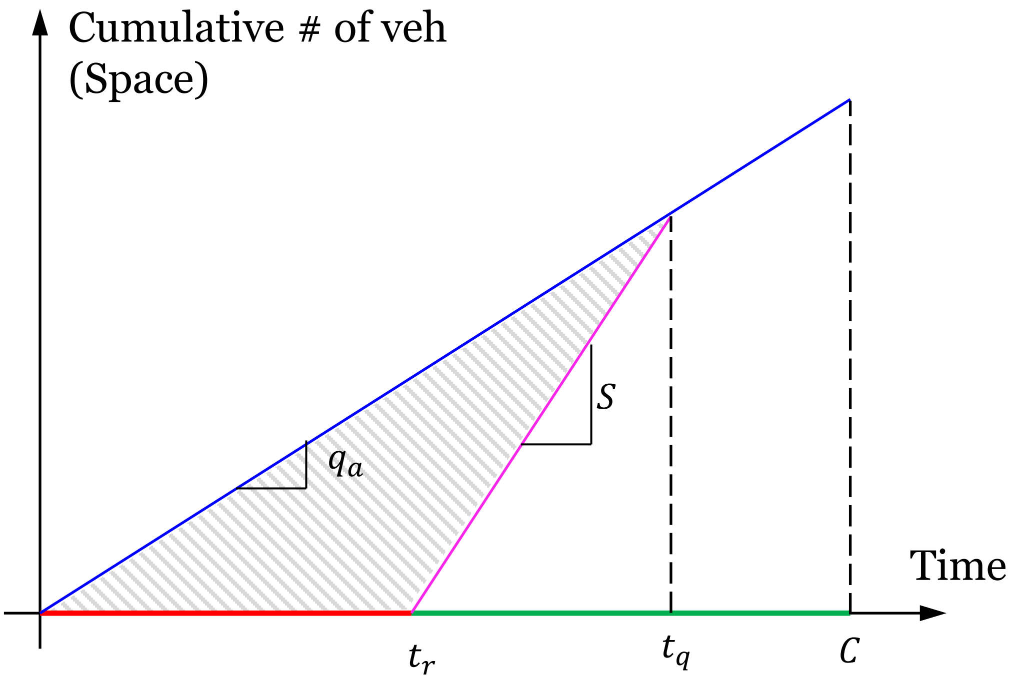
v/s ratio: $Y = \frac{q_a}{s}$

v/c ratio: $X = \frac{Y}{\theta}$
(Degree of saturation)

$$d = \frac{(1-\theta)^2 C^2}{2C \cdot (1 - X\theta)} = \frac{C(1-\theta)^2}{2 \cdot (1 - \theta X)}$$

Average delay per vehicle

Proportion of vehicles that must stop

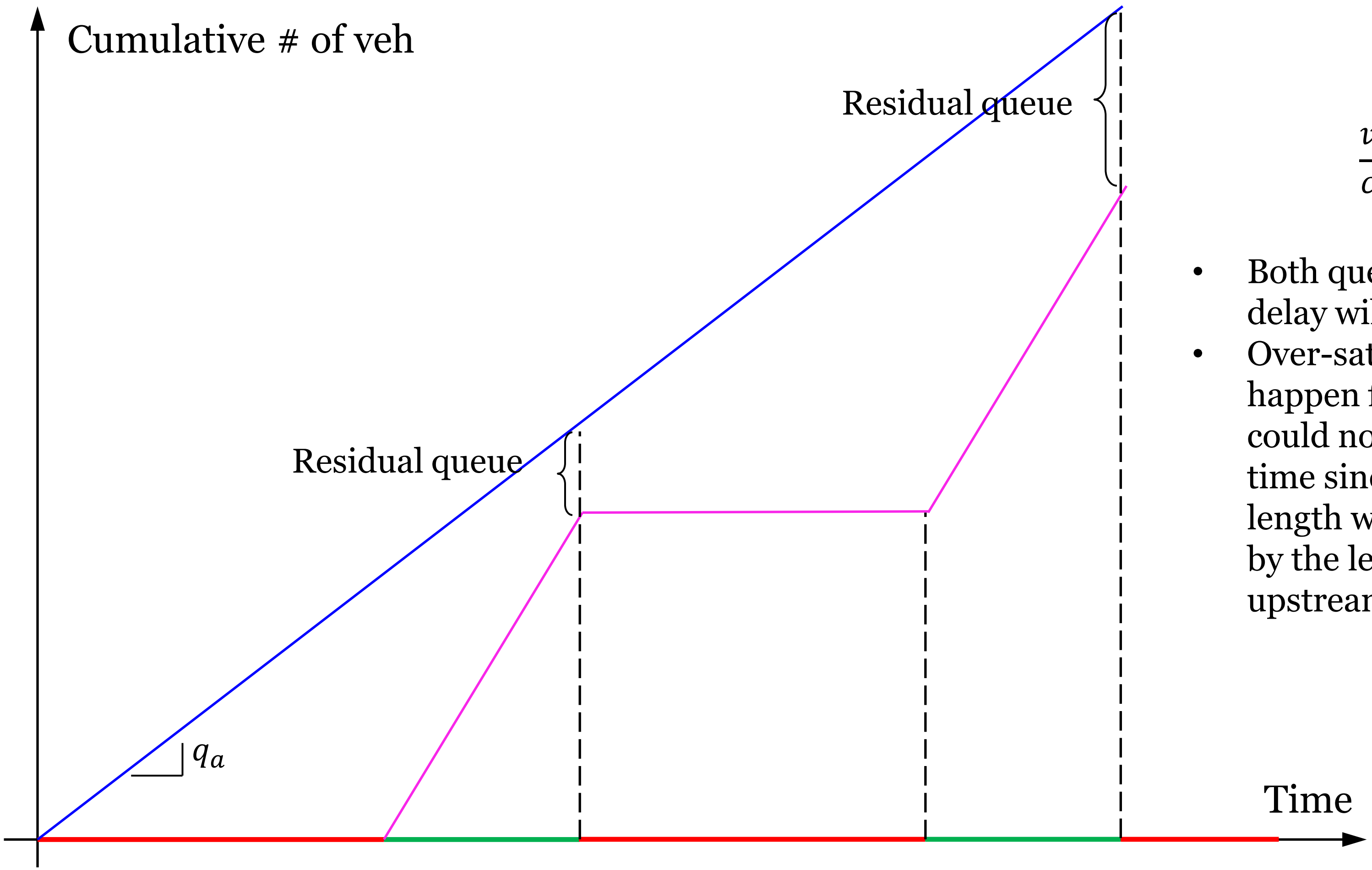


- Vehicles that arrive between $t = 0$ to $t = t_q$ must stop

$$\eta = \frac{1}{C} \cdot (t_r + t_q)$$

$$= \frac{t_r}{C \cdot (1 - Y)}$$

Over-Saturation



$$\frac{v}{c} > 1$$

- Both queue length and delay will be unbounded
- Over-saturation could happen for a while but could not happen a long time since the queue length will be bounded by the length of the upstream road

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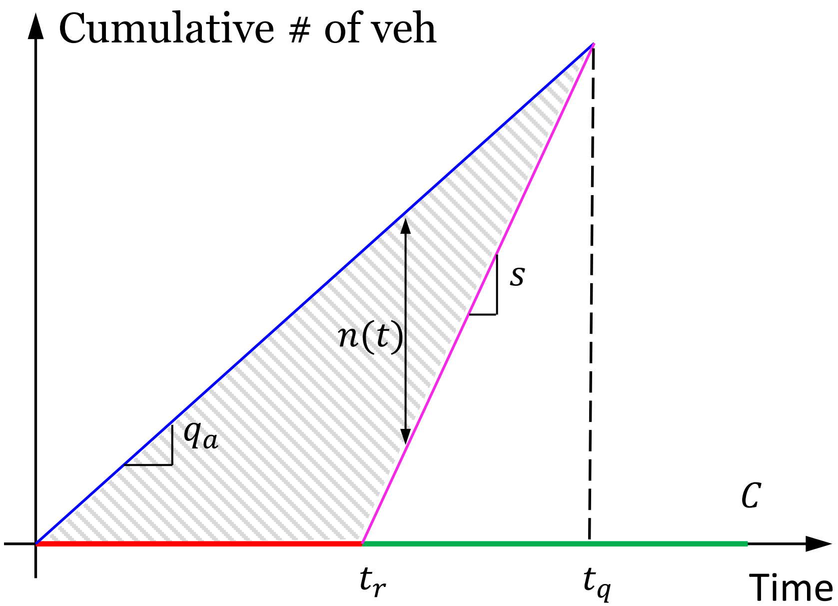
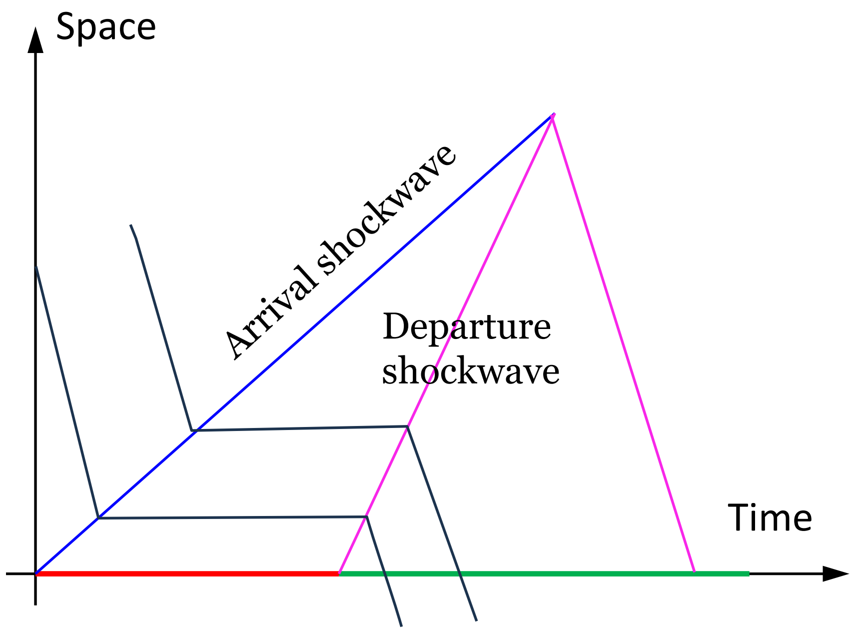
Queueing Diagram & TS Diagram



Queueing diagram

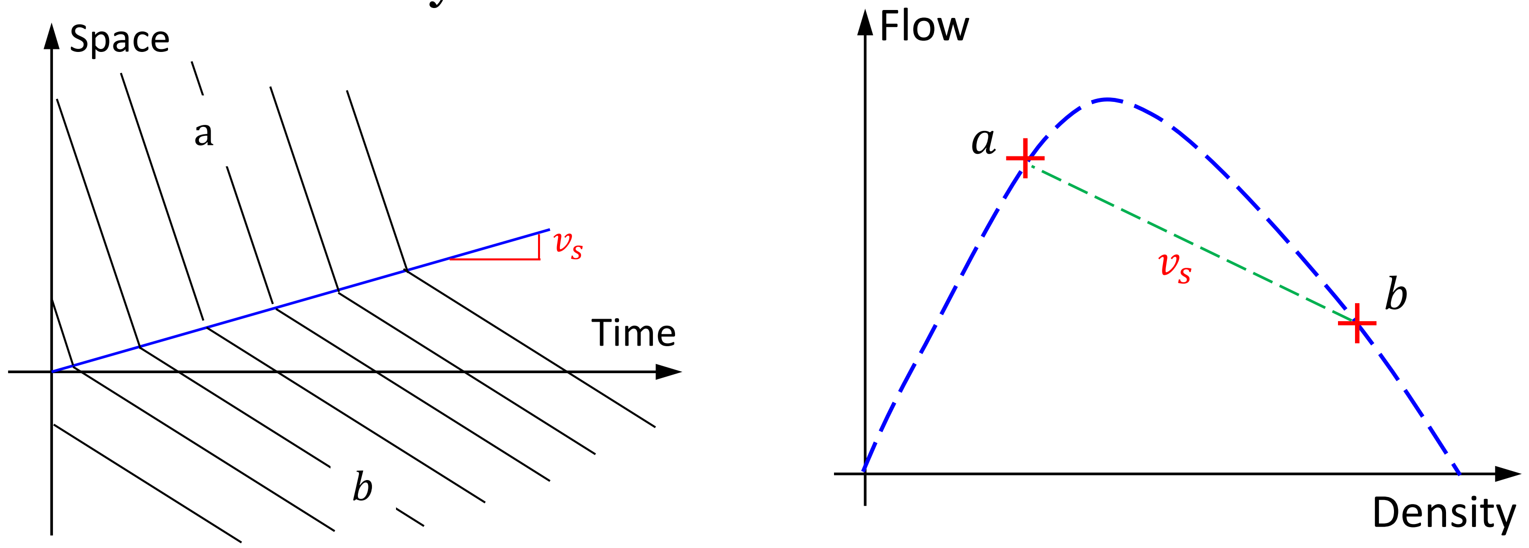


Time-space diagram and shockwave theory

We can also calculate delay using time-space diagram

Recap: Shockwave Theory

- Shockwave theory



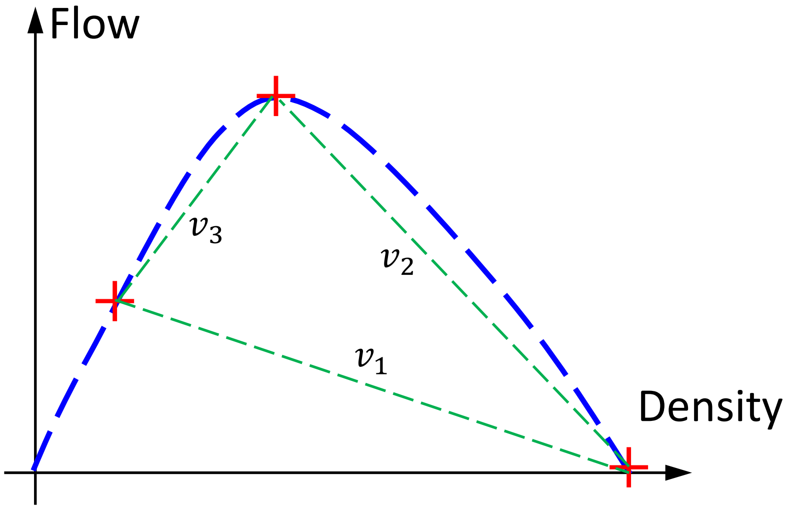
  - a, b: uniform traffic states
  - Shockwave: boundary between different traffic states
  - Shockwave speed

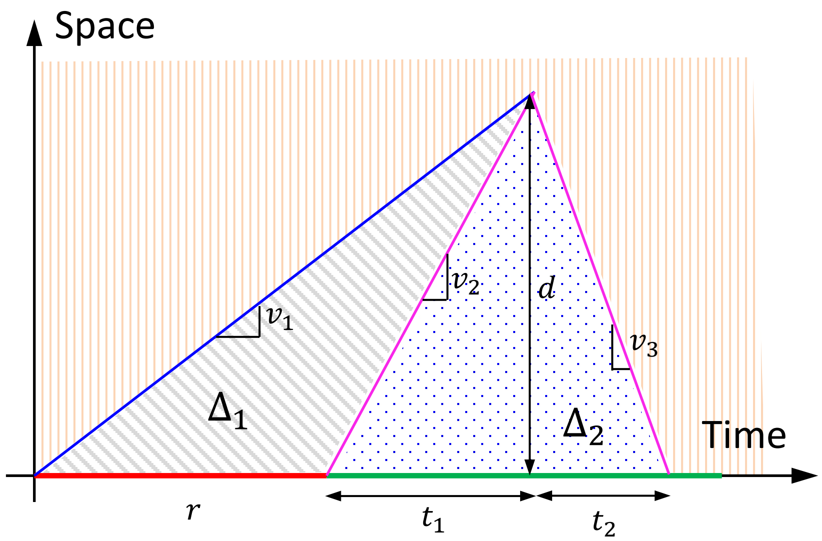
$$v_s = \frac{q_b - q_a}{k_b - k_a}$$

- $v_s > 0$ goes downstream
- $v_s < 0$ goes upstream

Time-Space Diagram

- Time-space diagram for signalized intersections

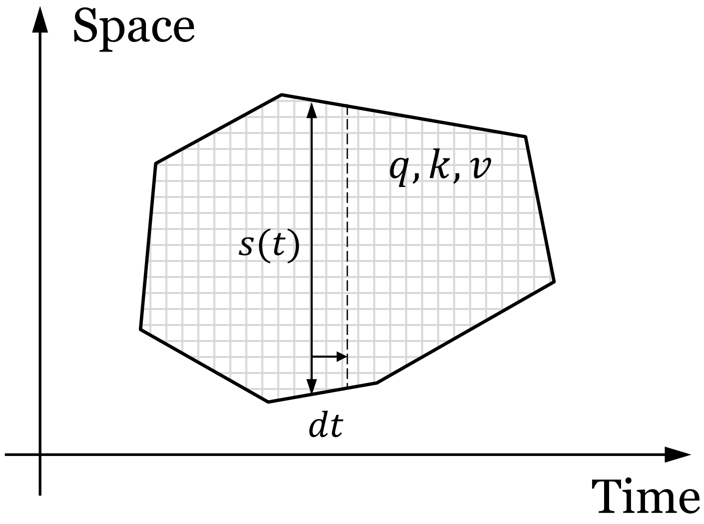




- How we calculate the total delay in TS diagram?

Total Travel Time in the Time-Space Diagram

- Obtain the travel time given the time-space diagram



- Number of vehicle in dashed area at time t

$$n(t) = s(t) \cdot k$$

- Total travel time from t to $t + dt$

$$n(t) \cdot dt$$

– What is the total travel time when the vehicle passing by the dashed area?

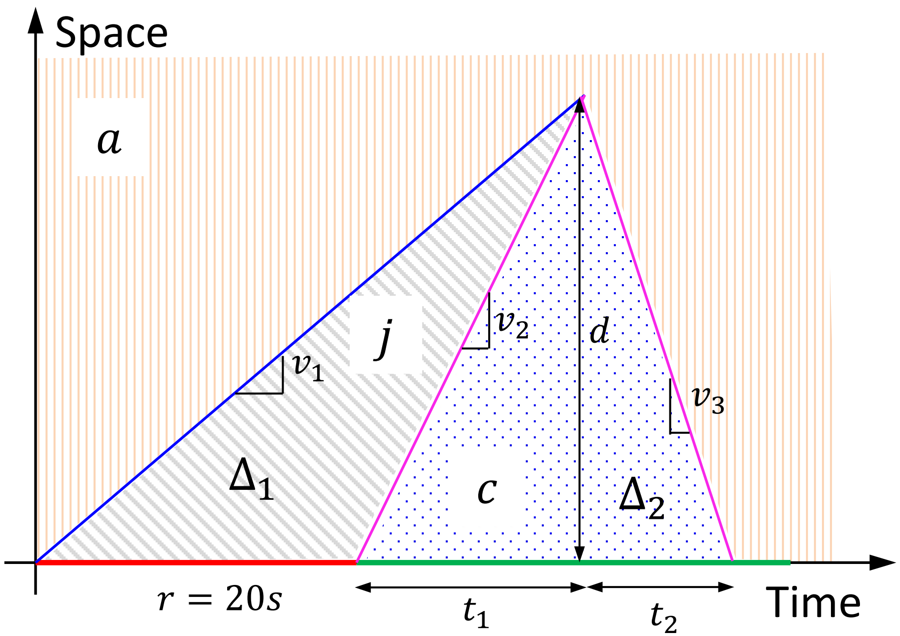
$$TTT = \int n(t)dt = k \int s(t)dt = k \cdot S_0$$

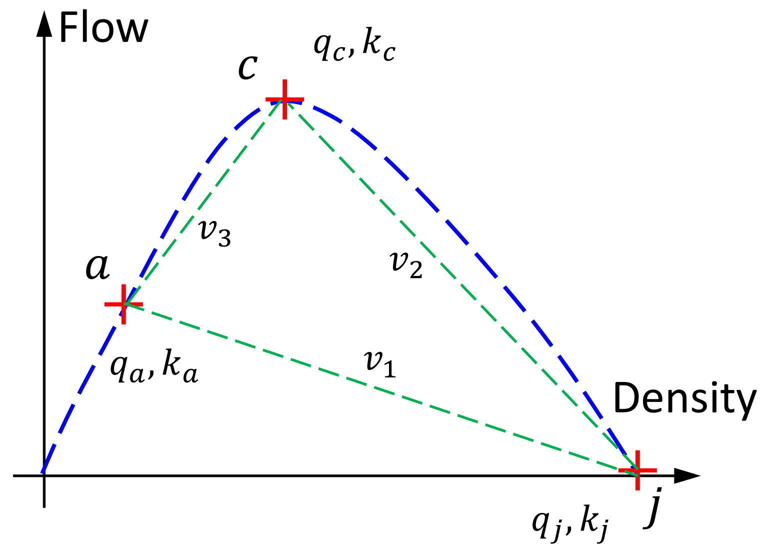
Check the unit!

– S_0: area in the time-space diagram

Total Delay

- Total delay for all vehicles



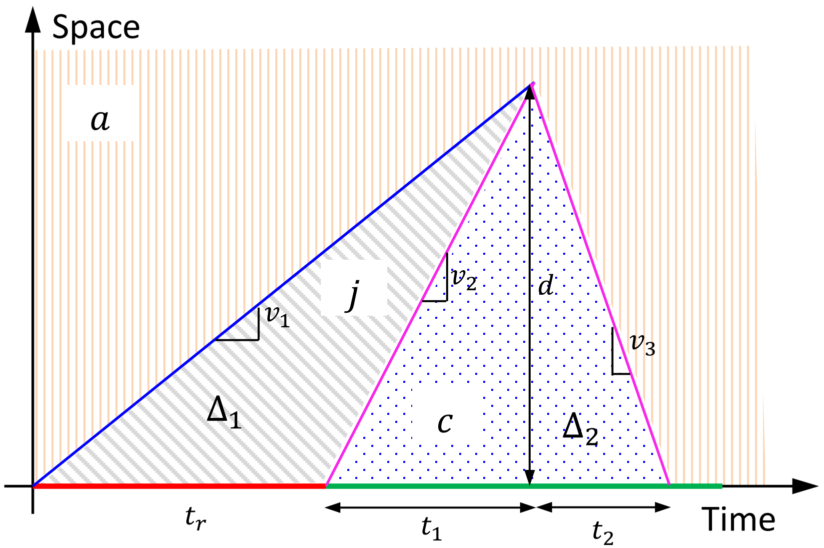


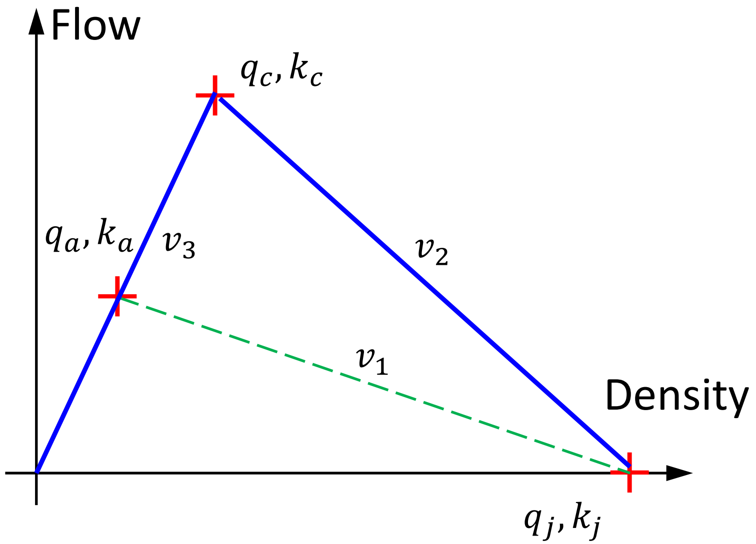
$$\text{Delay} = k_j \cdot \Delta_1 + k_c \cdot \Delta_2 - k_a(\Delta_1 + \Delta_2)$$

Total travel time with signal   Total travel time without signal

Triangular FD

- Equivalency between two delay models under triangular fundamental diagram





$$\text{Delay} = k_j \cdot \Delta_1 + k_c \cdot \Delta_2 - k_a(\Delta_1 + \Delta_2)$$

$$|v_1| = \frac{q_a}{k_j - k_a} \qquad |v_2| = \frac{q_c}{k_j - k_c}$$

$$|v_3| = v_f = \frac{q_a}{k_a} = \frac{q_c}{k_c}$$

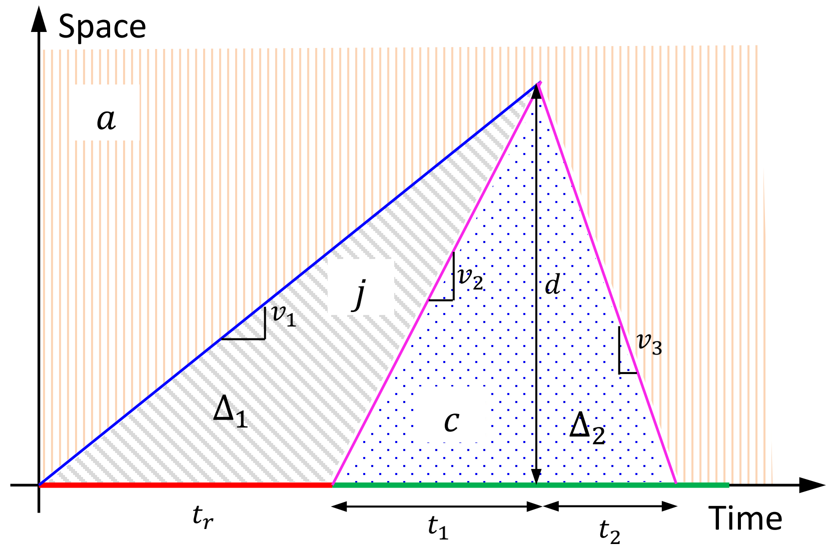
Total Delay Equivalency

$$\text{Delay} = k_j \cdot \Delta_1 + k_c \cdot \Delta_2 - k_a(\Delta_1 + \Delta_2)$$

$$(t_1 + t_r)|v_1| = |v_2|t_1 \qquad t_1 = \frac{t_r|v_1|}{|v_2| - |v_1|}$$

$$d = |v_2| \cdot t_1 = \frac{t_r|v_1 v_2|}{|v_2| - |v_1|}$$

$$\Delta_1 = \frac{1}{2} \cdot t_r\, d \qquad \Delta_2 = \frac{1}{2} \cdot (t_1 + t_2) \cdot d$$



$$k_c\Delta_2 = k_a(\Delta_1 + \Delta_2)$$

$$q_a(t_r + t_1 + t_2) = q_c(t_1 + t_2)$$

$$\frac{q_a}{k_a} = \frac{q_c}{k_c} \Rightarrow \frac{k_c}{k_a} = \frac{t_r + t_1 + t_2}{t_1 + t_2} = \frac{\Delta_1 + \Delta_2}{\Delta_2}$$

$$\text{Delay} = k_j\, \Delta_1 = \frac{1}{2} k_j \cdot \frac{t_r^2 |v_1 v_2|}{|v_2| - |v_1|}$$

$$= \frac{1}{2} k_j t_r^2 \cdot \frac{1}{1/|v_1| - 1/|v_2|}$$

$$= \frac{1}{2} k_j t_r^2 \cdot \frac{q_a q_c}{k_j(q_c - q_a)} = \frac{1}{2} t_r^2 \cdot \frac{q_a}{1 - q_a/q_c}$$

Why are they equivalent?

- Triangular fundamental diagram corresponds to the Newell's simplified car-following model
- In this case, the trajectory only has two states: 1) free-flow state and 2) stop state
- All delay are caused by the stop state (stop delay)
- A simple queueing diagram that only considers vehicle stop is sufficient to calculate the total delay

- If the FD is not triangular, they will be different

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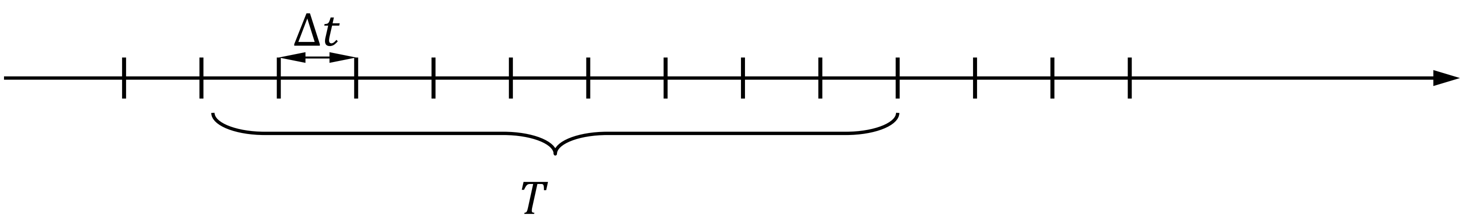
Random Arrival & Poisson Distribution

- In the real world, the number of arrival is random
- For example, for each cycle (100 seconds), the average arrival of a certain movement is 8 veh
- For a specific cycle, the actual number of arrival could be different (it is a random variable)
- The number of arrival for each cycle is usually modeled by a **Poisson distribution**

Bernoulli Arrival & Poisson Distribution

- A brief derivation of the Poisson distribution



  - For each time interval, there could be an arrival with probability p (Bernoulli distribution)
  - Let n be the total intervals within the horizon T and a is the total number of arrivals

$$n = \frac{T}{\Delta t} \qquad a = a(\Delta t) + \cdots + a(n\Delta t) \implies a \sim \text{Binomial}(n, p)$$

  - Average number of arrival: $\mathbb{E}(a) = \sum_{i=1}^{n} a(i\Delta t) = np = \lambda$
  - When $\Delta t \to 0, n \to \infty$: $a \sim \text{Poisson}(\lambda)$

Poisson Distribution

- Total number of arrivals

$$a \sim \text{Binomial}(n, p) \qquad \lambda = np$$

Average number of arrivals (bounded constant)

$$\mathbb{P}(a = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$n \to +\infty$$

$$\log(1-p)^{n-k} = (n-k)\log\left(1 - \frac{\lambda}{n}\right) \to (n-k)\left(-\frac{\lambda}{n}\right) \to -\lambda$$

(Taylor's expansion)

$$\mathbb{P}(a = k) = \frac{n!}{k!\,(n-k!)} \cdot \frac{\lambda^k}{n^k} e^{-\lambda} = \frac{n!}{n^k (n-k!)} \cdot \frac{\lambda^k}{k!} e^{-\lambda} \to \frac{\lambda^k e^{-\lambda}}{k!}$$

$$\mathbb{P}(a = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

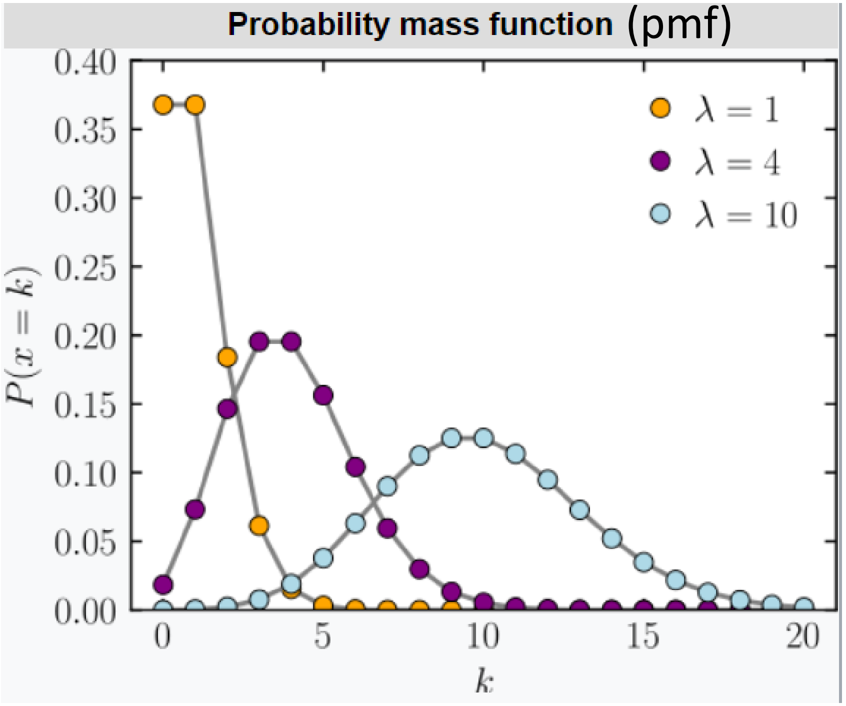
Poisson distribution

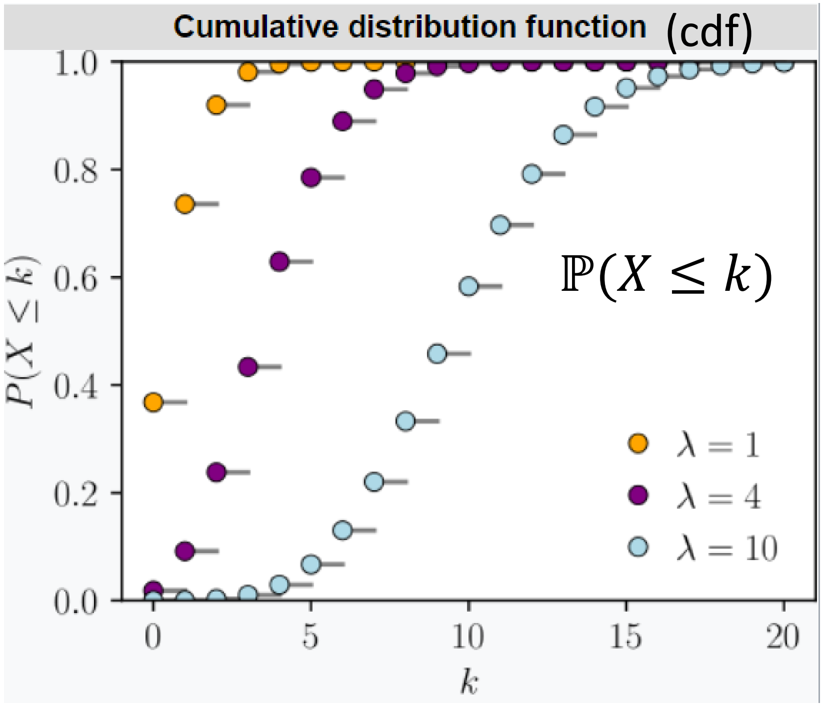
Poisson Distribution

- Average number of arrival λ

$$\mathbb{P}(a = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

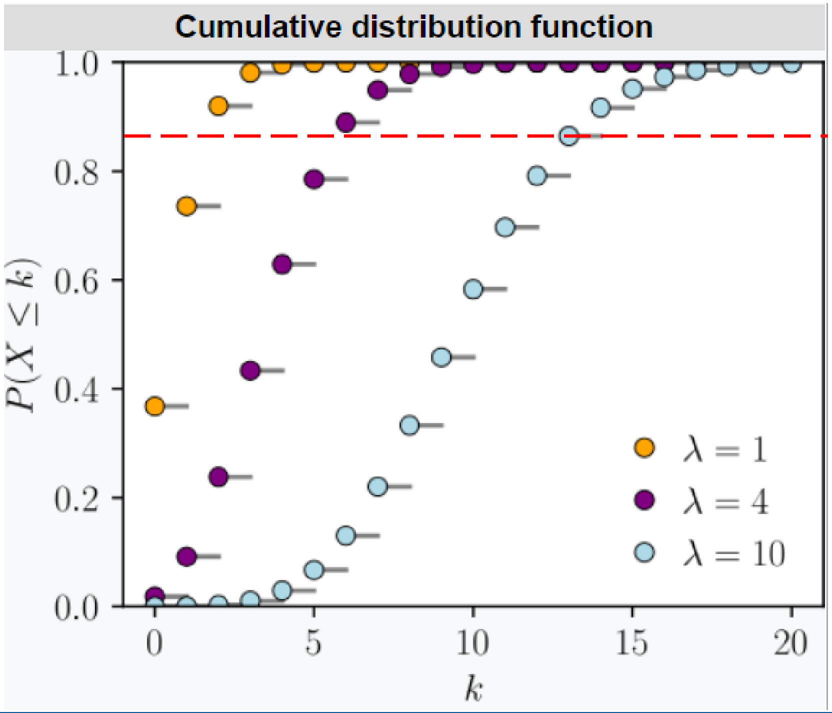
Poisson distribution (probability mass function)





Poisson Distribution & Maximum Green

- In last lecture, we discussed about the traffic signal design, the green duration & cycle length calculation is based on the average number of arrivals
- For the maximum green time calculation, we can follow the same procedure but use 85th percentile (or as requested) of the Poisson distribution as the actual volumes



Basic Queueing Models

- Birth-death model (discrete model)

  - Discrete Markov model with arrival probability λ (birth) and departure probability μ (death)
- Basic queueing model (continuous model, $\Delta t \rightarrow 0$)
  - M/M/1 queue: stochastic arrival (Poisson process λ) and stochastic departure (exponential service time)
  - M/D/1 queue: stochastic arrival (Poisson process λ), deterministic departure (fixed service time $1/\mu$)

Queueing Models

- For both queueing models, key parameters include the arrival rate λ, and departure rate μ
- Utilization rate $\rho = \frac{\lambda}{\mu}$, $\rho < 1$, under-saturated, $\rho = 1$, saturated, $\rho > 1$, over-saturated
- For under-saturated case $\rho < 1$, the stochastic queueing model will have a stationary distribution (system has an average state over time)

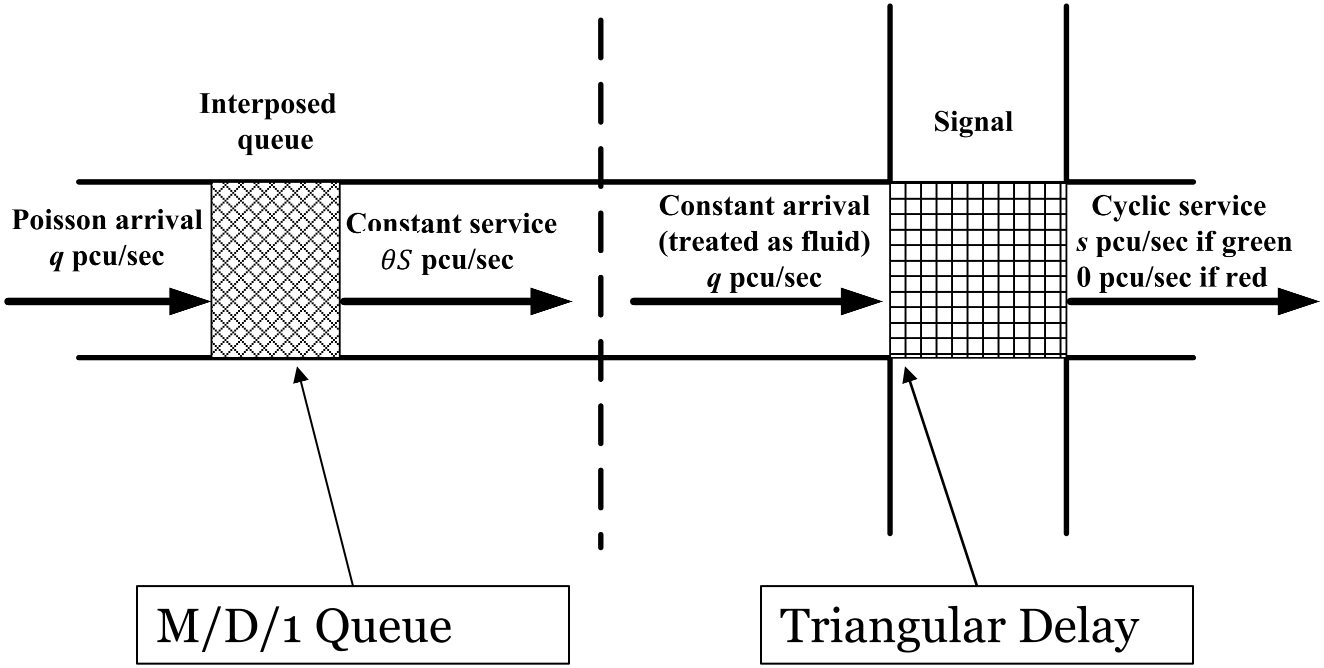
Comparison of M/D/1 and M/M/1 queue properties

	M/D/1	M/M/1
E(w) (average waiting time)	$E(w) = \frac{\rho}{2\mu(1-\rho)}$	$E(w) = \frac{\lambda}{\mu(\mu-\lambda)}$
E(v) (average total delay)	$E(v) = \frac{2-\rho}{2\mu(1-\rho)}$	$E(v) = \frac{1}{(\mu-\lambda)}$
E(n) (expected number of units in the system (including vehicles being served))	$E(n) = \frac{(2-\rho)\rho}{2(1-\rho)}$	$E(n) = \frac{\rho}{1-\rho}$

Total delay = waiting time + service time

Delay by Random Arrivals

- Divide vehicles passing the intersection by two steps (Allsop, 1972):



Delay by Random Arrivals

- M/D/1 Queue:

$$X = \frac{Y}{\theta} = \frac{q}{S\theta}$$

Average length of queue in vehicles:

$$\bar{Q} = \frac{X^2}{2(1-X)}$$

Average waiting time in the queue:

$$\bar{w} = \frac{X}{2\theta S(1-X)} = \frac{X^2}{2q(1-X)}$$

Average time spent in the system:

$$\bar{t} = \frac{2X - X^2}{2q(1-X)}$$

X: degree of the saturation
q: arrival rate
θ: green split

Webster's Delay

- Webster's delay equation:

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + \frac{x^2}{2q(1-x)} - 0.65\left(\frac{c}{q^2}\right)^{\frac{1}{3}} x^{(2+5\lambda)}$$

- Optimal cycle length:

$$C_{\text{opt}} = \frac{1.5 \times L + 5}{1.0 - Y_c}$$

- Optimal effective green split:

$$g_i = \frac{q_i/S_i}{\sum_{j=1}^{n} q_j/S_j} g_t$$

where, $g_t = C - L$ is the total green time.

Other Delay Models

- Webster's delay is one of the delay model
- There are other delay models or empirical equations
  - Akcelik Delay Model
  - HCM (Highway Capacity Manual) 2000 delay models (https://en.wikibooks.org/wiki/Fundamentals_of_Transportation/Traffic_Signals)

Readings

- Basic queueing models https://en.wikibooks.org/wiki/Fundamentals_of_Transportation/Queueing
- Delay models: https://www.civil.iitb.ac.in/tvm/nptel/572_Delay_A/web/web.html#x1-270005.2