CEE 551 - Traffic Science

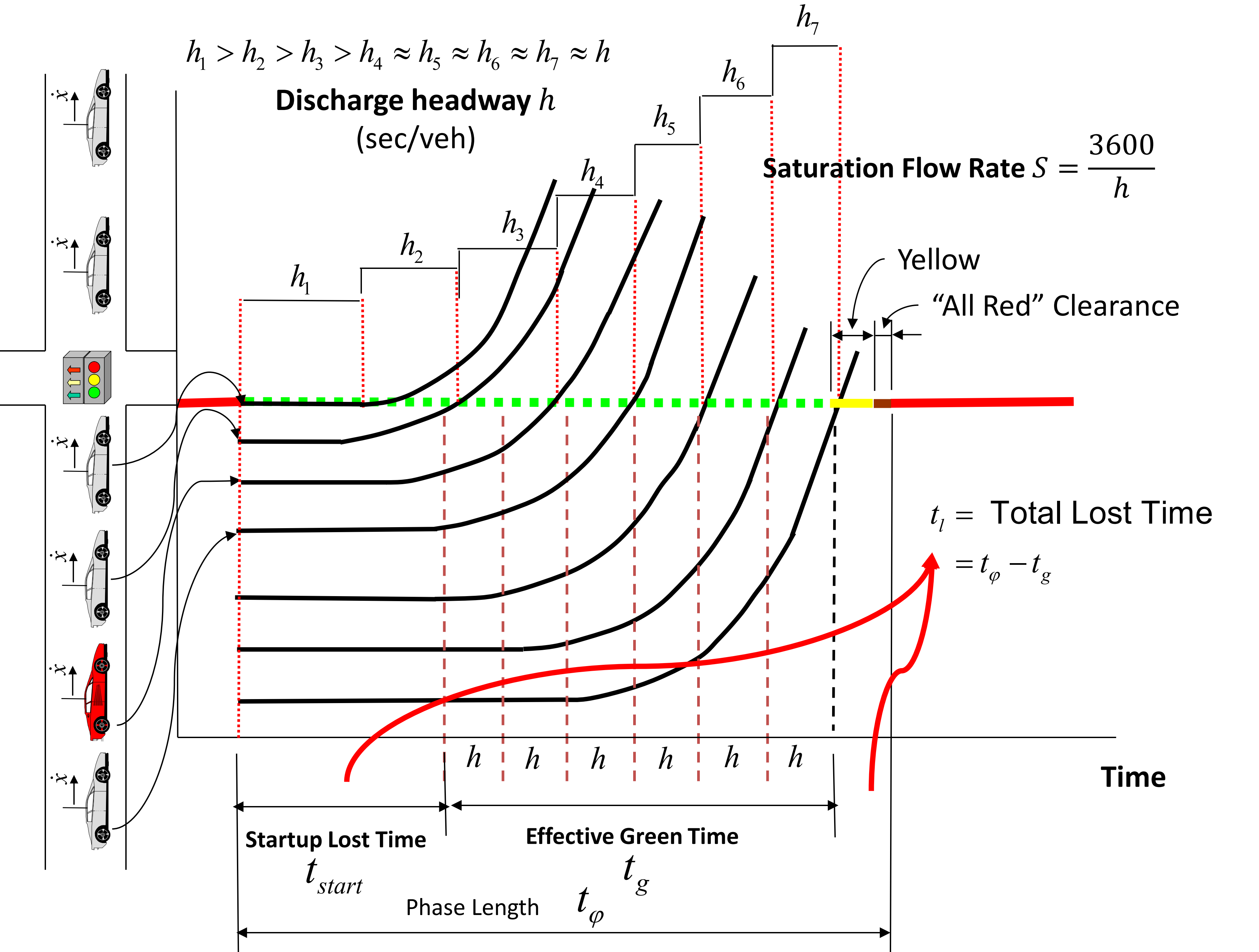
Topic: Traffic Signal Control (3)

Xingmin Wang

Department of Civil and Environmental Engineering
University of Michigan
Email: xingminw@umich.edu

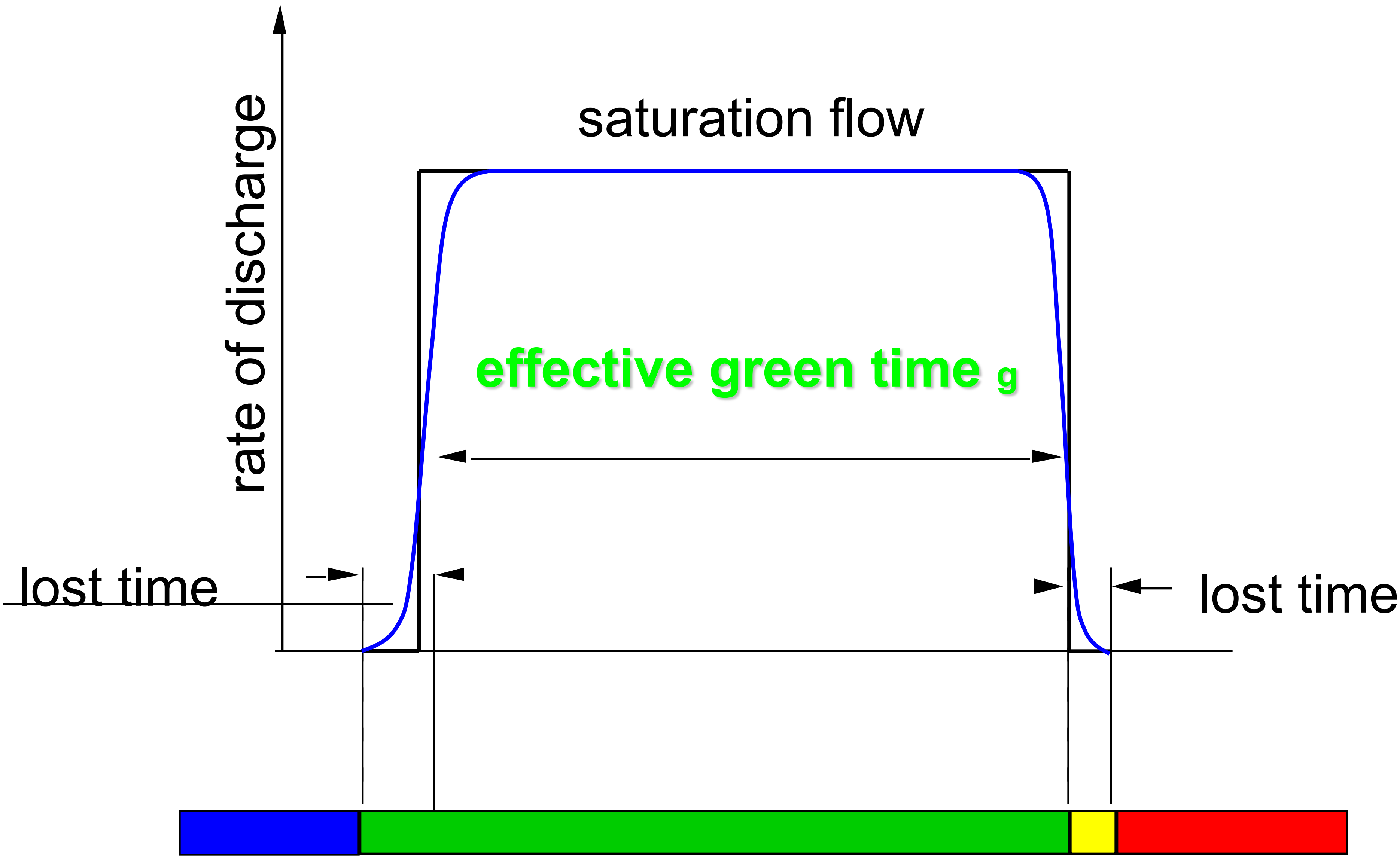
Signal Timing Design

- Some basic concepts
  - Saturation flow rate, effective green, and loss time
  - v/s ratio and v/c ratio
  - Degree of saturation for intersections
- Critical movement analysis
- Signal timing design
- Example
- Delay analysis (level of service)



$h_1 > h_2 > h_3 > h_4 \approx h_5 \approx h_6 \approx h_7 \approx h$
Discharge headway h
(sec/veh)
h_1
h_2
h_3
h_4
h_5
h_6
h_7
Saturation Flow Rate $S = \frac{3600}{h}$
Yellow
"All Red" Clearance
$t_l =$ Total Lost Time
$= t_\varphi - t_g$
h h h h h h
Time
Startup Lost Time
t_{start}
Effective Green Time
t_g
Phase Length t_φ

Effective Green



Loss Time

- For each phase i, we have the loss time:

$$G_i + Y_i + AR_i = L_i + g_i$$

  - G_i: display green
  - Y_i: yellow change interval
  - AR_i: all-red clearance time
  - L_i: loss time
  - g_i: effective green

Comments:

- Loss time includes the start-up loss and end-of-green loss
- Not all yellow and all-red are loss time (roughly less than half of the yellow plus the all-red)

Per-Lane Volume & v/s ratio

- Per-lane volume: total traffic volume divided by the number of lane
- For each movement, the v/s ratio is defined as the ratio of the traffic volume to the saturation flow rate for all lanes (per-lane volume divided by the saturation flow rate per-lane)
- Saturation flow rate is the maximum flow in FD, determined by the road condition including speed limit, lane width, etc.
- Saturation flow rate per lane is usually a constant (e.g., 1800 veh/(hour*lane)), therefore, per-lane volume & v/s ratio are almost equivalent

$$v/s \text{ ratio} = \text{per-lane volume} \;/\; \text{saturation flow rate (per lane)}$$

- v/s ratio is more rigorous than per-lane volume considering the different saturation flow rate

v/s Ratio and Green Split (Ratio)

- Let g_i be the effective green of movement i, the green ratio is defined as:

$$\text{Green ratio of movement } i = \frac{g_i}{C}$$

- The (effective) green time ratio should be larger than the v/s ratio of the movement, otherwise, the green time cannot satisfy the traffic demand

$$\frac{g_i}{C} \geq \frac{v_i}{s_i}$$

- Example: for a movement with 1 lane, saturation flow rate 1800 vph, average traffic volume 600 vph. v/s ratio is 1/3. This means that, for each cycle, we need the effective green is larger than 1/3 of the cycle length

v/s and v/c Ratio

- For a certain movement or phase

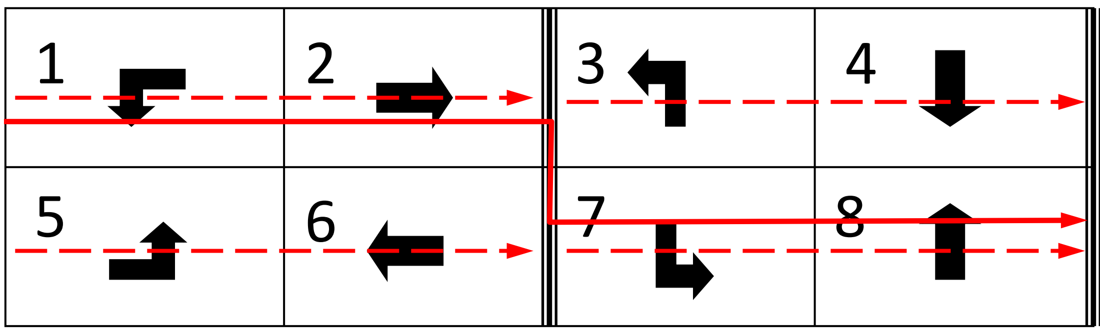
$$\theta_i = \frac{g_i}{C} \qquad Y_i = \frac{v_i}{s_i} \qquad X_i = \frac{v_i}{c_i} = \frac{v_i}{s_i \cdot \theta_i} \le 1 \qquad X_i = \frac{Y_i}{\theta_i}$$

Green ratio   v/s ratio   v/c ratio or Degree of saturation

  - v_i: traffic volume of movement i
  - θ_i: green ratio
  - g_i: effective green time
  - C: cycle length
  - c_i: capacity of the movement

Critical Movement Analysis

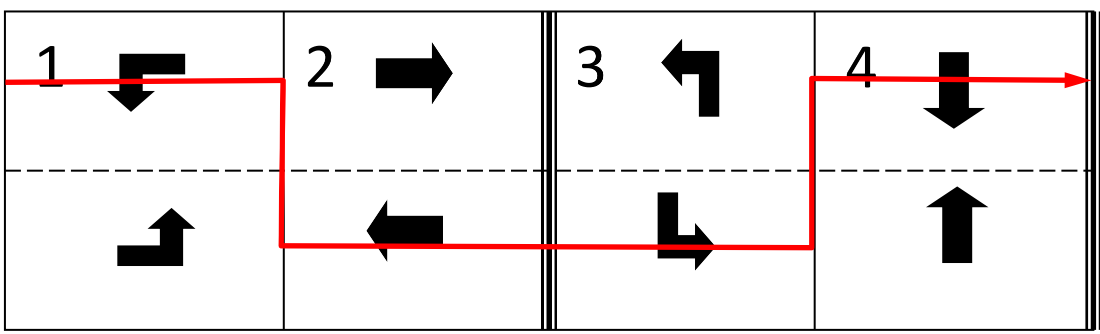
- "Critical path" in the ring-and-barrier diagram



(more flexible)

$$\max\{v_1 + v_2, v_5 + v_6\} + \max\{v_3 + v_4, v_7 + v_8\} = K_1$$

$$K_1 \leq K_2$$

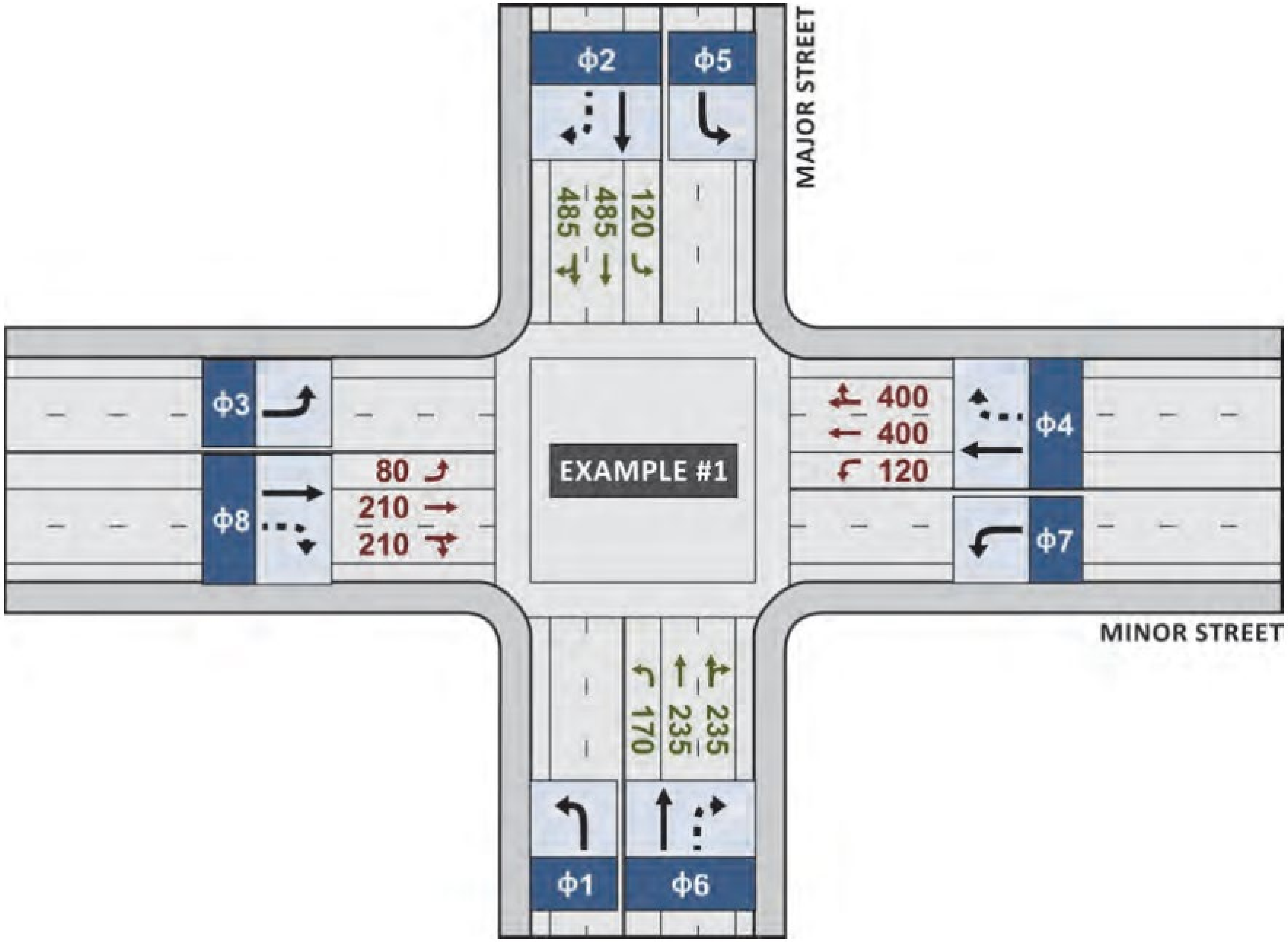


(more commonly used)

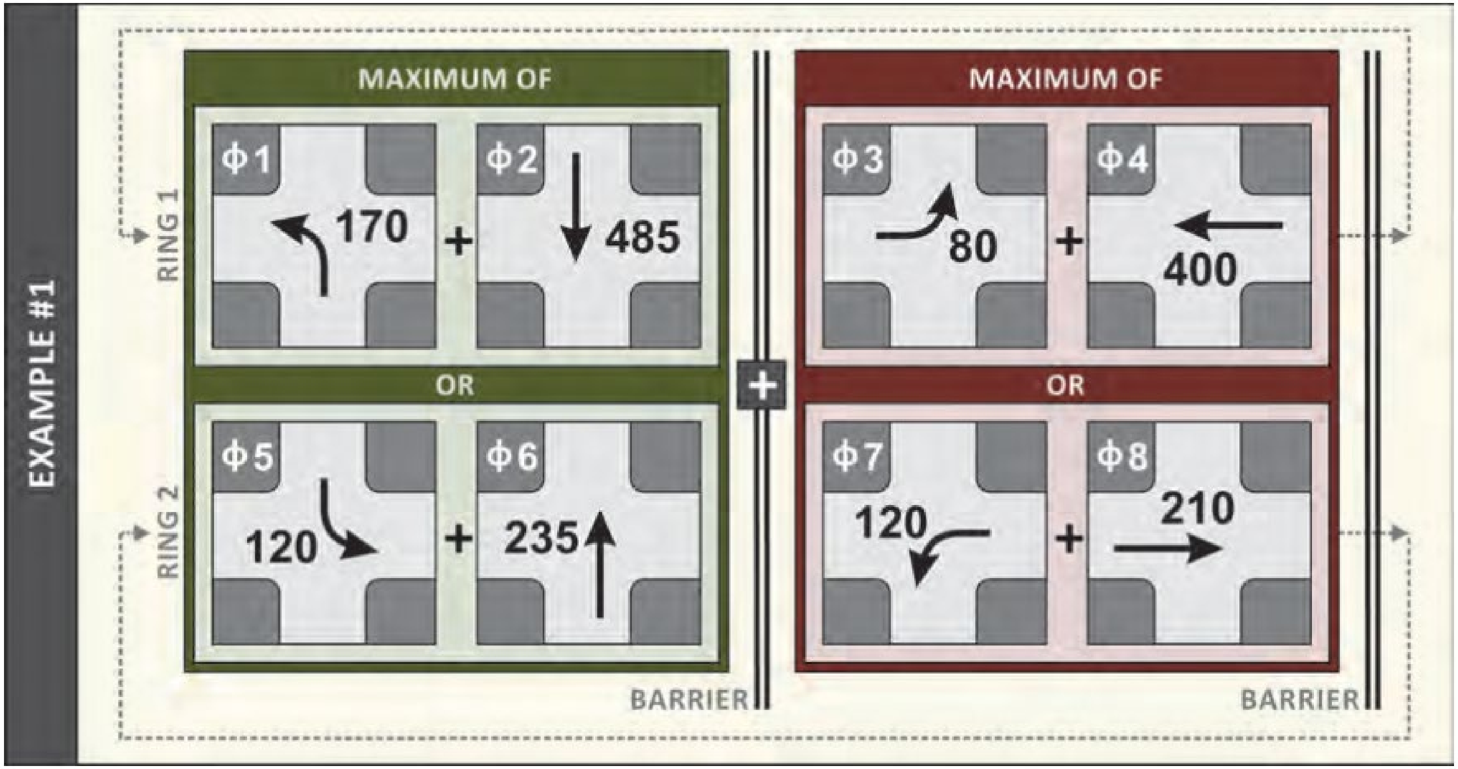
$$\max\{v_1, v_5\} + \max\{v_2, v_6\} + \max\{v_3, v_7\} + \max\{v_4, v_8\} = K_2$$

(Single-ring operation)

Critical Movement Analysis



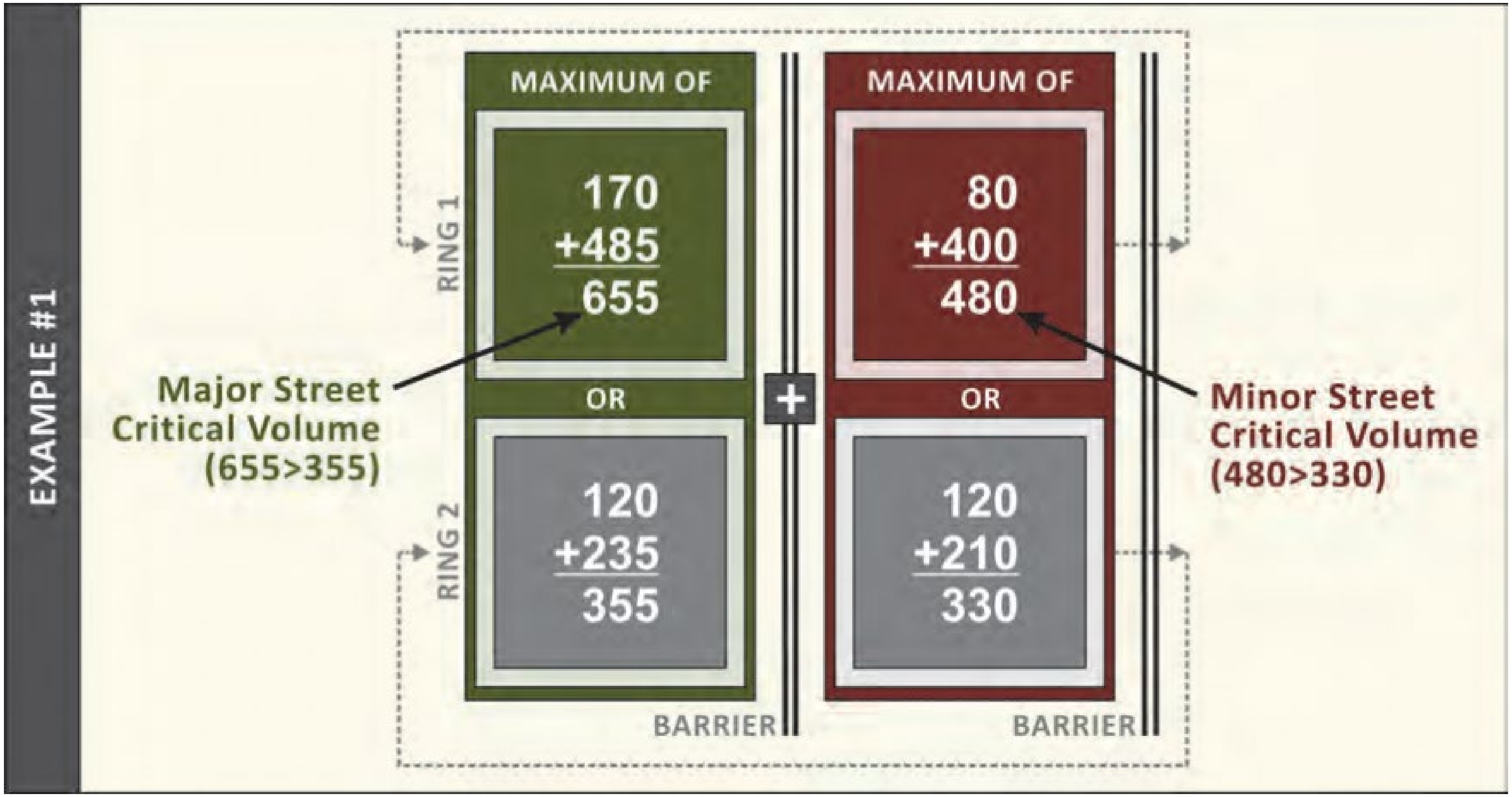
Determine the Critical Movement



$v_1 + v_2 = 170 + 485$ $\quad$ $\frac{v_1}{s_1} + \frac{v_2}{s_2} = \frac{170 + 485}{1800} = 0.36 \leq \frac{g_1 + g_2}{C}$

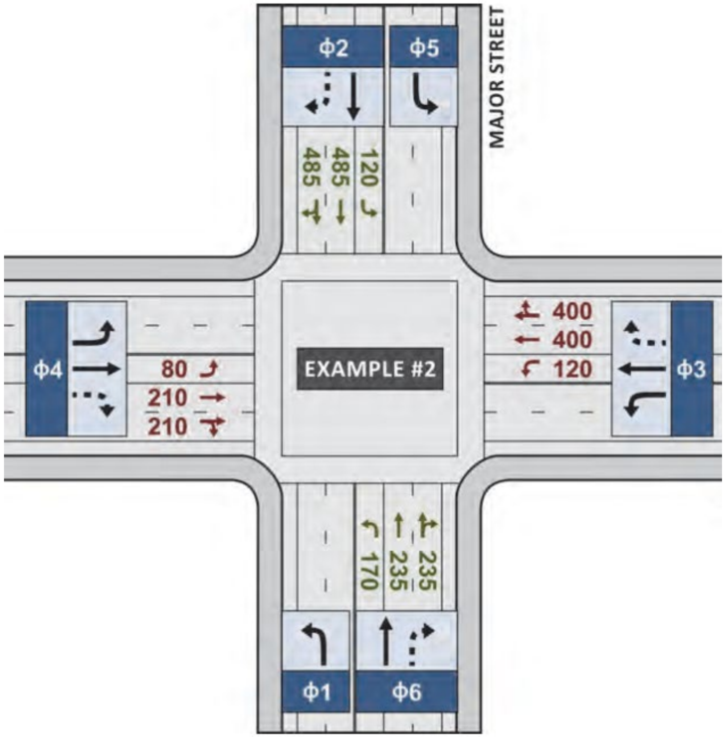
(Critical v/s ratio)

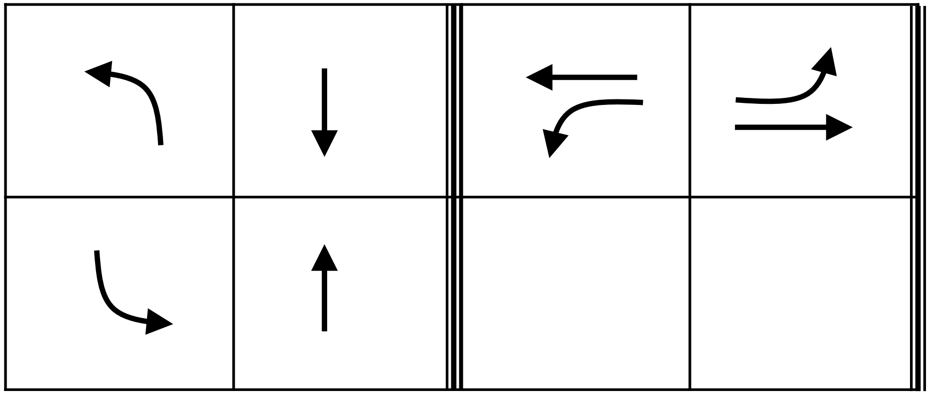
Critical Movement (Per-Lane) Volume



Exercise: Critical Movement Analysis

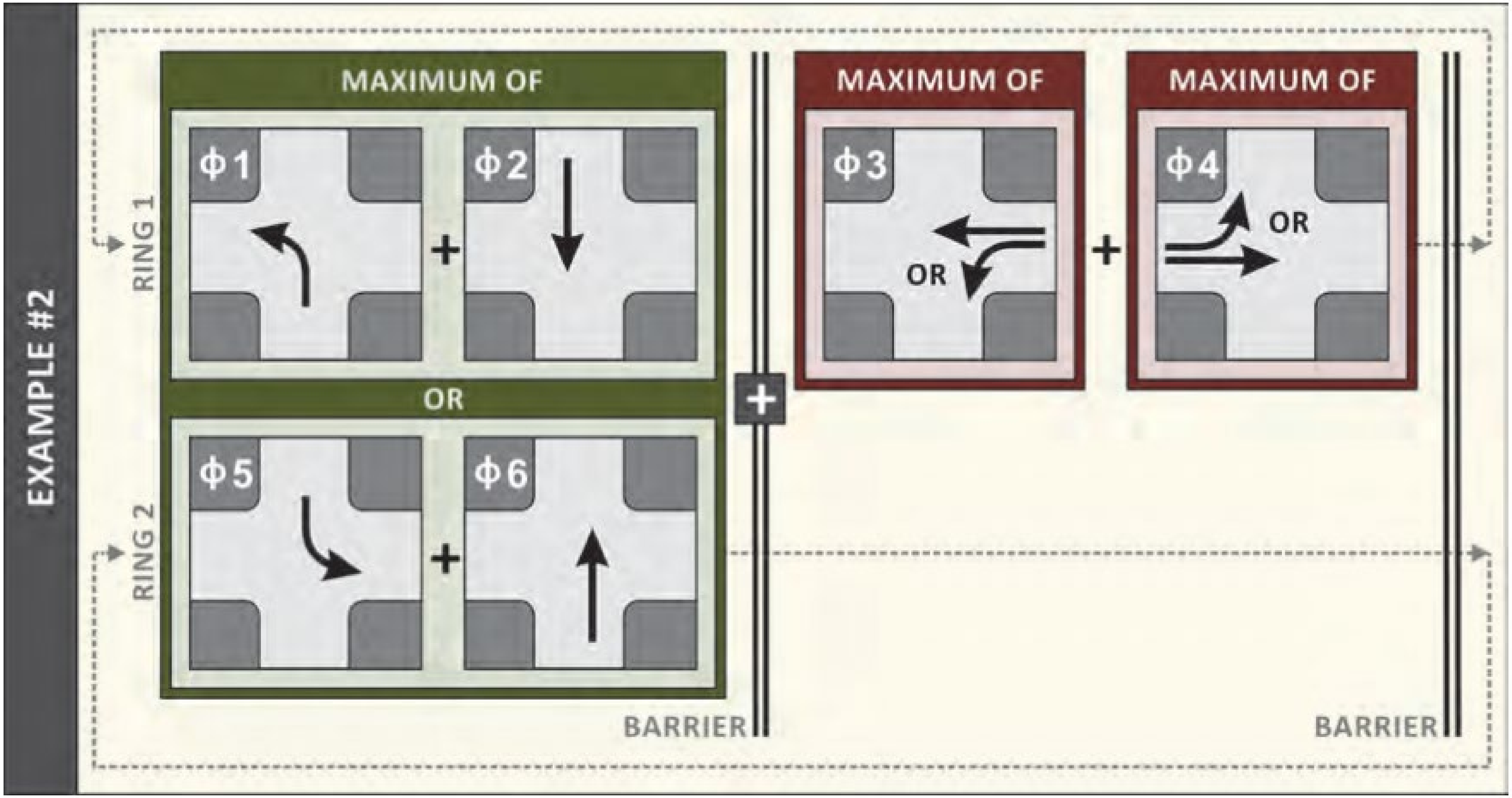
- What if we use a different phase structure?



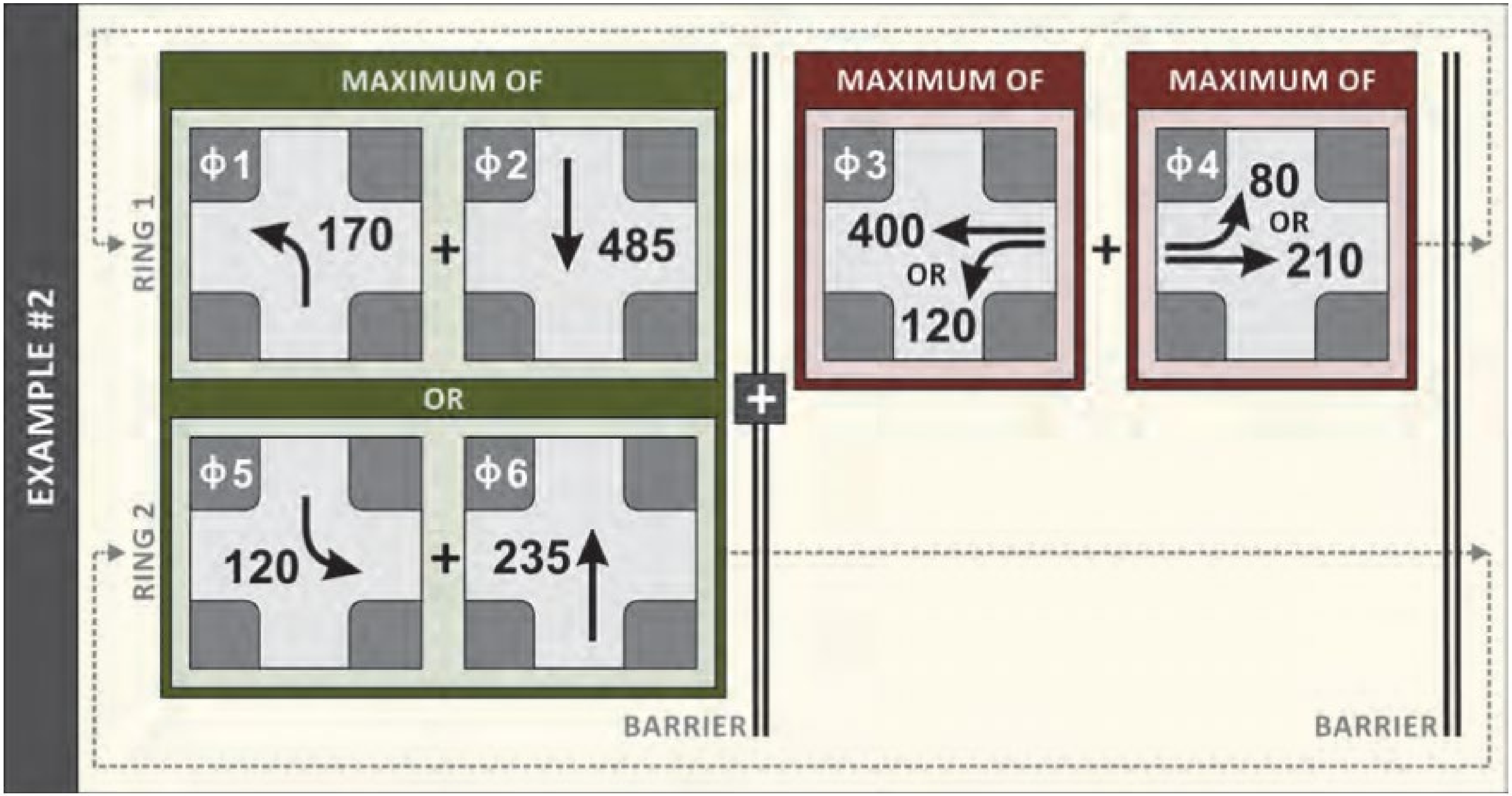


Split-phasing for east-west direction

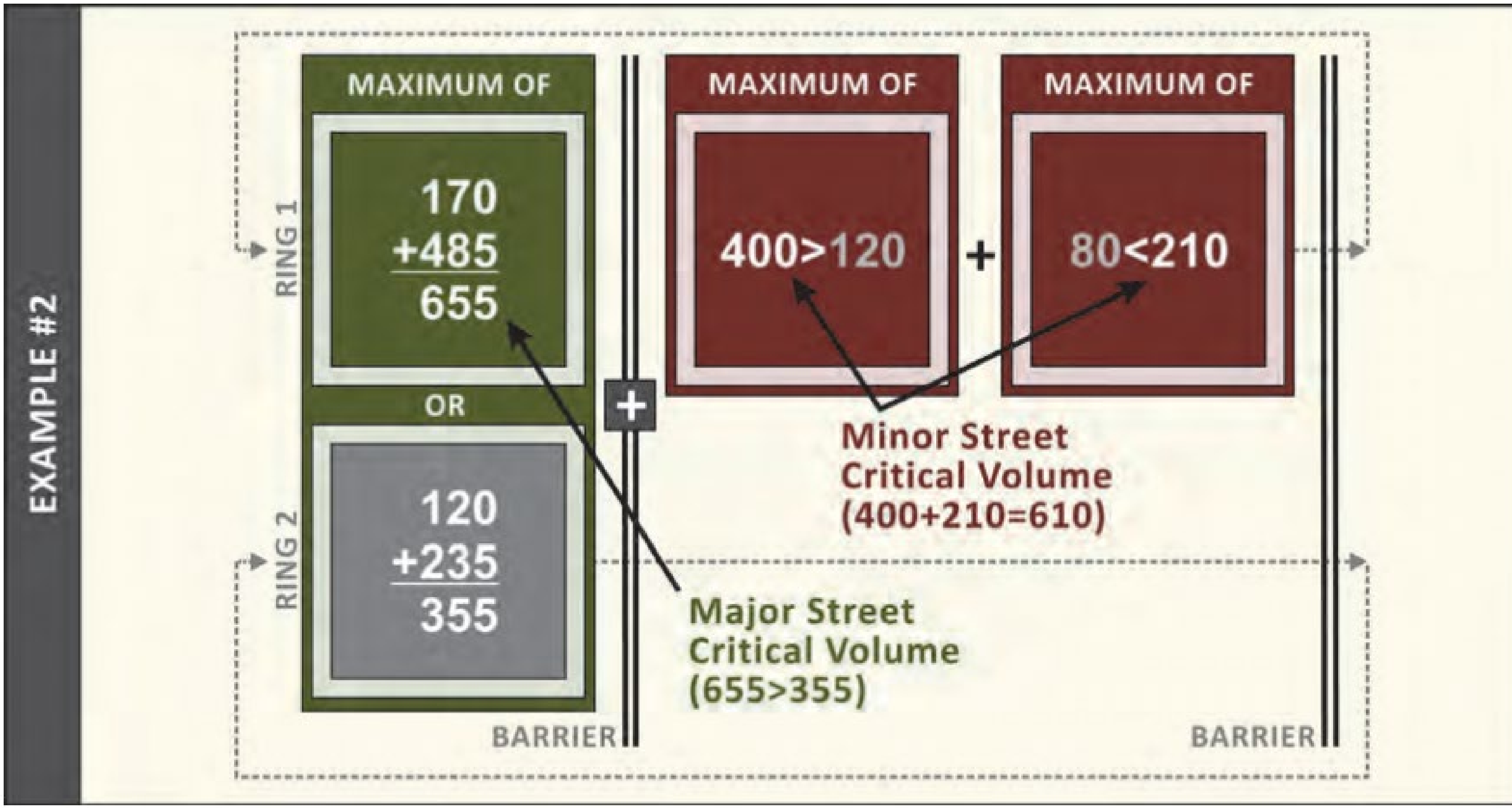
Critical Movement Analysis



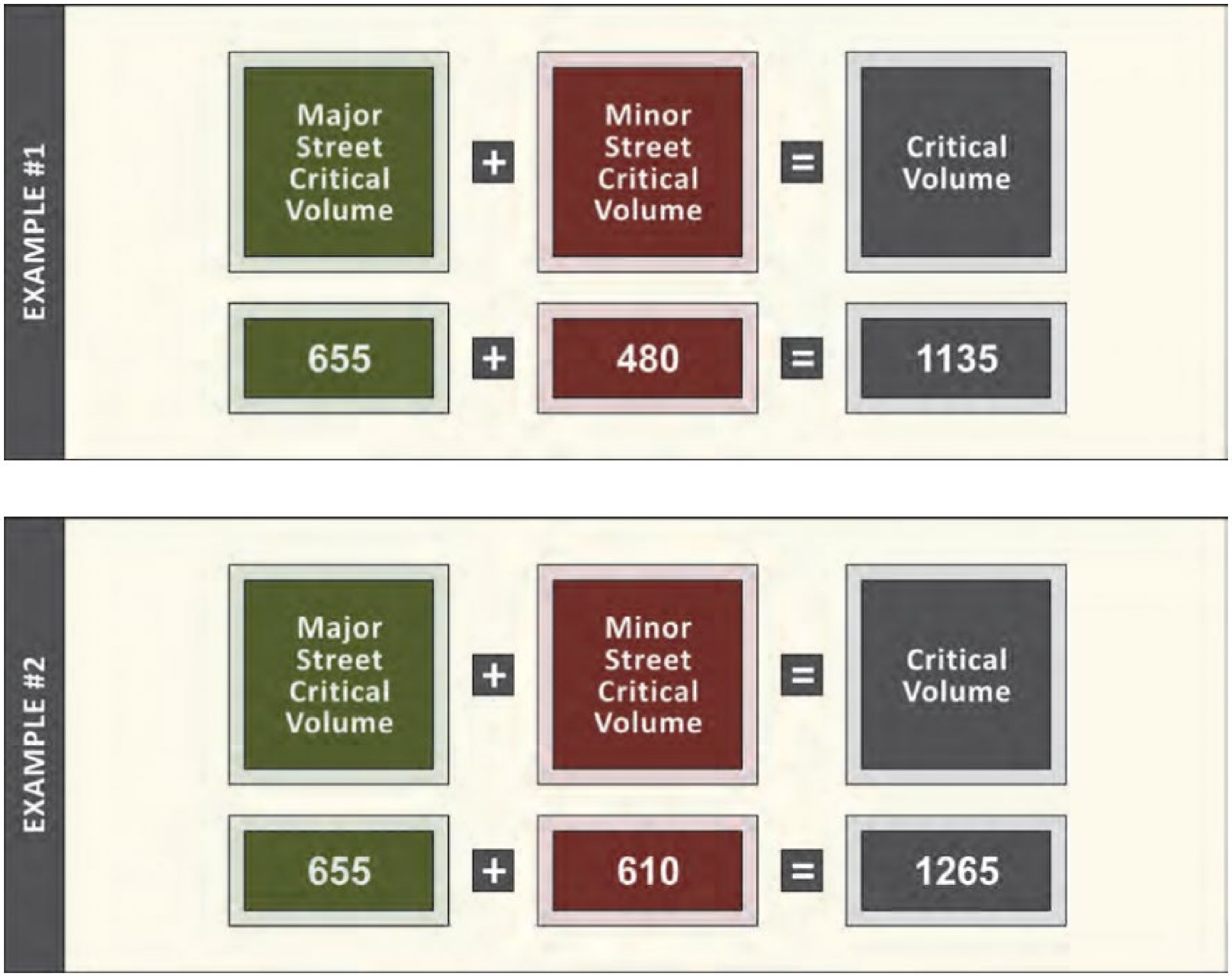
Determine Critical Movement



Critical Movement Volume



Comparison



- Which is better?
- Different phase structure could lead to different critical per-lane volumes

Degree of Saturation for an Intersection

- Degree of saturation for an intersection
  - Essentially the volume-to-capacity ratio for intersections
  - Minimum required effective green ratio divided by the total available effective green ratio

$$Y_c = \sum_i Y_i = \sum_i \frac{v_i}{s_i} \qquad X_c = \frac{Y_c}{\frac{1}{C} \cdot (C - L)} = \frac{Y_c \times C}{C - L}$$

    - Y_c: summation of the v/s ratio along the critical path
    - L: total loss time in a cycle
    - C: cycle length
  - Monotonically decreasing with the increase of C

Critical-Movement Volumes and Cycle Lengths

- The critical (per-lane) volume that can be handled in 3600 seconds

$$V_C = s \cdot \left(1 - \frac{N t_L}{C}\right) = s\left(1 - \frac{L}{C}\right)$$

  - V_c: max critical-movement volumes
  - t_L: loss time per phase
  - N: number of phases in each cycle

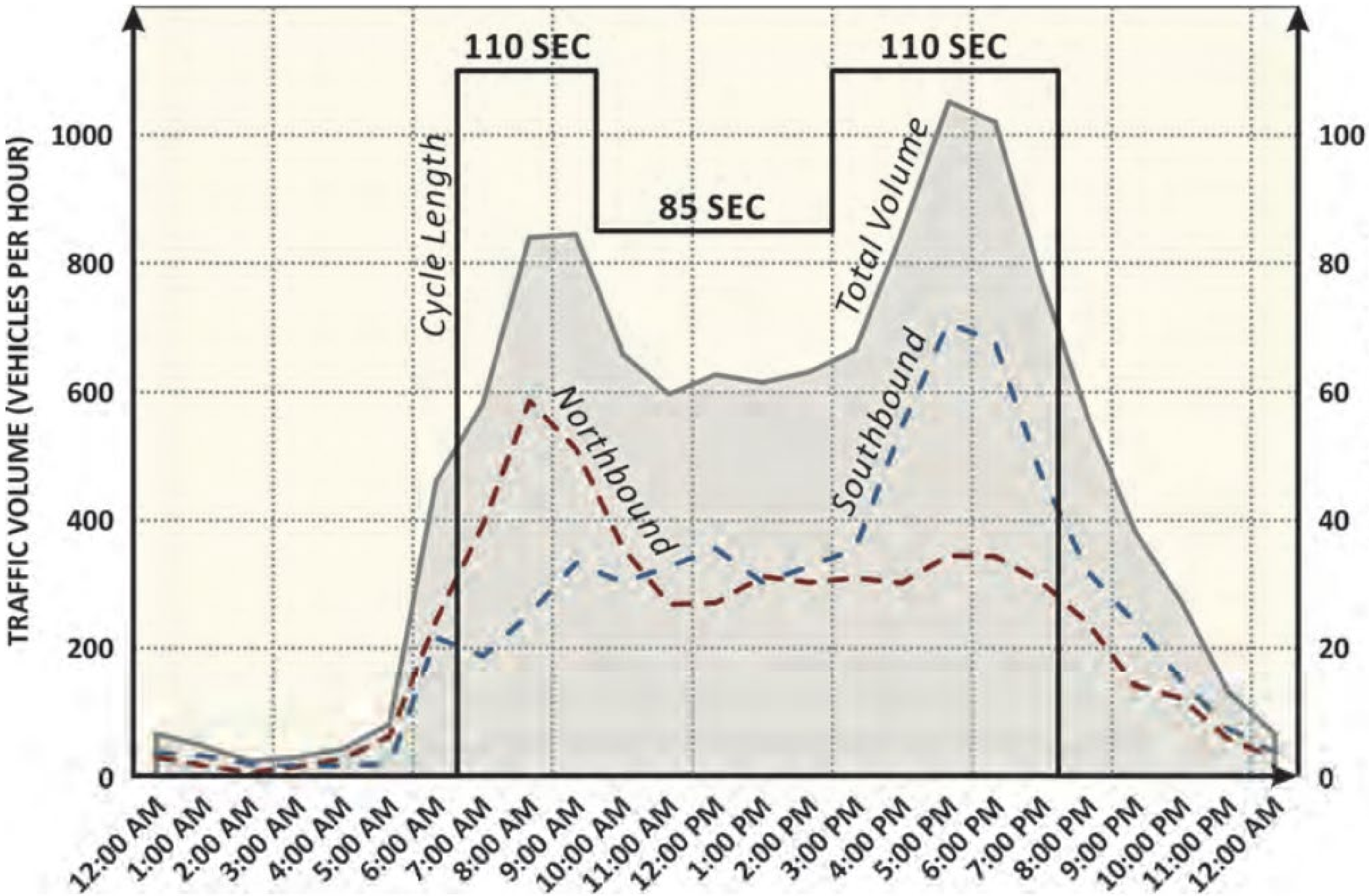
TABLE 20-3 Sum of critical-lane volumes for various *C* and *N* values

Cycle Length (sec)	*Number of Phases* *N* = 2	*N* = 3	*N* = 4
60	1507	1423	1340
80	1549	1486	1423
100	1574	1524	1473
120	1591	1549	1507

h = 2.15 sec/veh t_L = 3 sec/phase

How does total delay change with cycle length?

Cycle Length



Signal Timing Procedure (Steps)

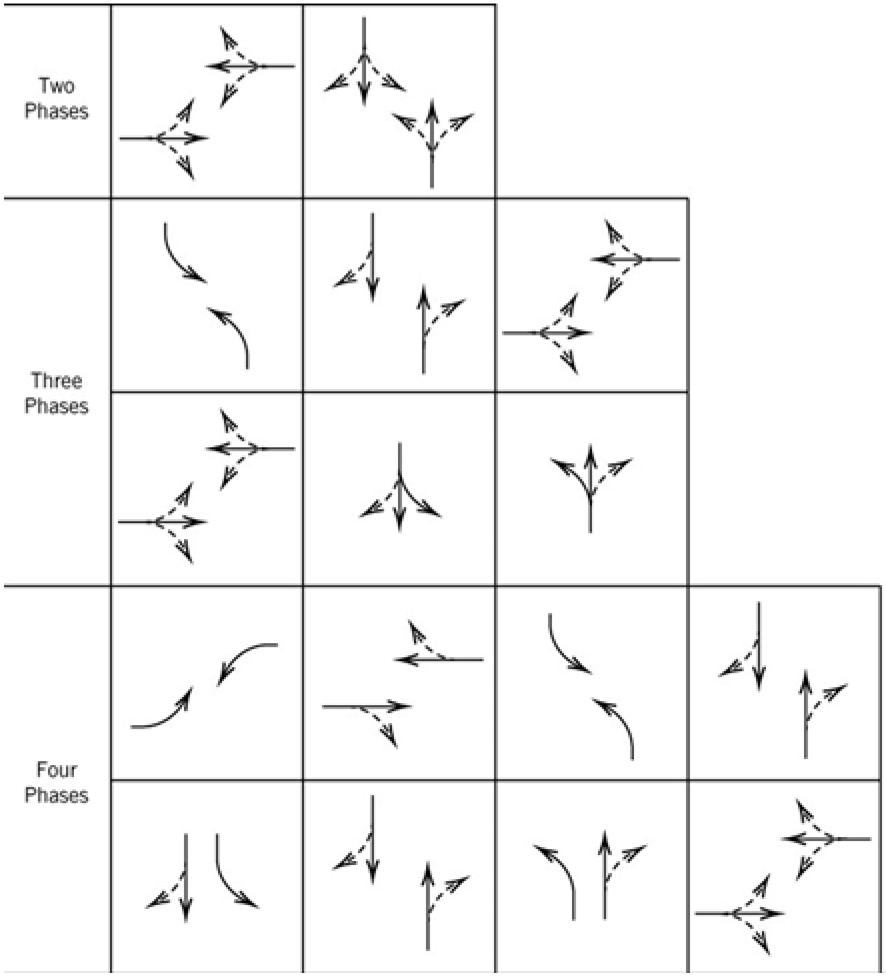
1. Determine "Phasing" (Ring-and-Barrier Diagram)
2. Analyze lane group
3. Critical Movement (Lane Group) Analysis
4. Calculate Cycle Length
5. Determine Yellow & Clearance Intervals
6. Determine Proportion of Green Time
7. Check Pedestrian Crossing Time
8. Prepare Signal Indication Diagram

Step 1: Determine Phases to Use

Two Phases

Three Phases

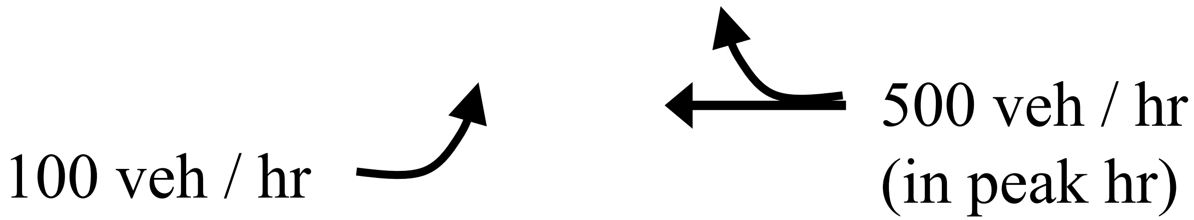
Four Phases



Different Sequences (single-ring)

Step 1: Determine Phases to Use

- Left Turn protected phase should be considered if any of the following criteria is met:
  - More than one turning lane is provided;
  - The left turn has a demand over 240veh/h;
  - The cross product of left turn demand and opposing through demand for 1 hour exceeds 50,000 for one opposing lane, 90,000 for two opposing through lanes, or 110,000 for three or more



Step 2: Analyze Lane Groups

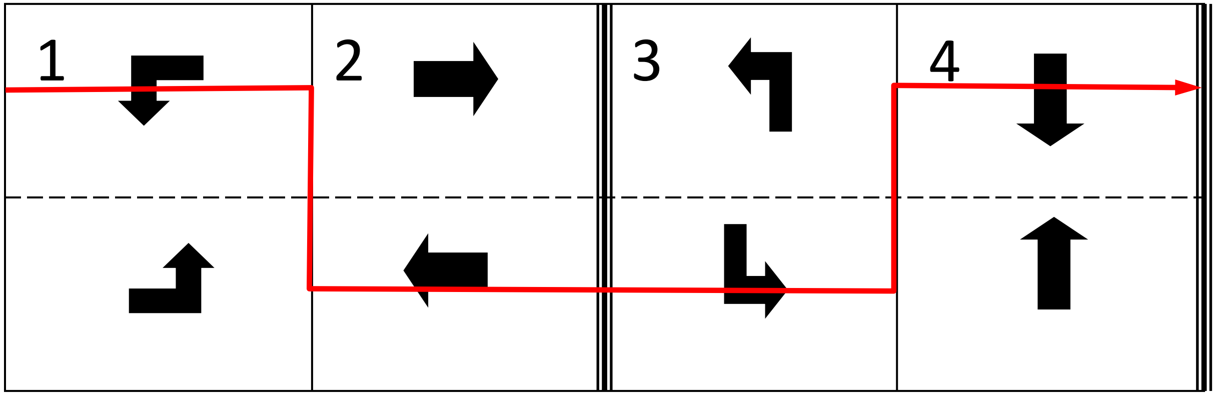
- Segmenting the intersection into lane groups
  – Geometry of the intersection
  – Traffic volume of different movements
- Lane group classification
  – Exclusive turn lane
  – Shared lane
  – Remaining through lane
- Step 1 & Step 2 design principles
  – Safety: minimize the conflicting among movements
  – Efficiency: minimize the critical movement volume (v/s ratio)

Step 2: Analyze Lane Groups

Number of lanes	Movements by lane	Number of possible lane groups
1	LT + TH + RT	① (Single-lane approach)
2	EXC LT TH + RT	②
2	LT + TH TH + RT	① or ②
3	EXC LT TH TH + RT	② or ③

Step 3: Critical Movement (Lane Group) Analysis

- Critical lane group (movement): the lane group with highest v/s ratio
- Allocation of green time for each phase is based on the v/s ratios



Single ring operation

Step 3: Determine Critical Movement (Lane Group)

- Summation of total critical v/s ratio:

$$Y_c = \sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} \left(\frac{v}{s}\right)_i$$

  - Y_c : sum of flow ratio for critical lane groups
  - $Y_i = \left(\frac{v}{s}\right)_i$: flow ratio for critical lane i
  - n: number of critical lane groups
- Total cycle lost time:

$$L = \sum_{i=1}^{n} (t_L)_i$$

  - L = Total lost time for cycle in seconds
  - $(t_L)_i$ = total lost time for critical lane group i in seconds
  - n = number of critical lane groups.

Step 4: Calculate Cycle Length

Recap: $X_c = \frac{Y_c \times C}{C - L}$

- Minimal necessary cycle length:

$$C_{\min} = \frac{L \times X_c}{X_c - Y_c}$$

  - $C_{\min}$ = minimum necessary cycle length in seconds (rounded up to the nearest 5s increment in practice)
  - X_c = (desired) critical v/c ratio of the intersection

- Optimal Cycle length (Webster, 1958)

$$C_{\text{opt}} = \frac{1.5 \times L + 5}{1.0 - Y_c}$$

  - C_{opt} = cycle length to minimize delay in seconds

Step 5: Green Time Allocation

- The effective green time is assigned to different phases proportional to the v/s ratio
- Total effective green time: $G = C - L$
- Effective green for each phase: $g_i = G \times \frac{v_i/s_i}{\sum_i v_i/s_i} = G \cdot \frac{Y_i}{Y_c}$

$$g_i = (C - L) \cdot \frac{Y_i}{Y_c} = \frac{Y_i \cdot C}{C \times Y_c/(C - L)} = Y_i \cdot \frac{C}{X_c}$$

Recap: $X_c = \frac{Y_c \times C}{C - L}$

Step 5: Allocate Green Time

- Recalculate X_c since the cycle length is rounded:

$$X_c' = \frac{Y_c \times C}{C - L}$$

- Green time for each phase:

$$g_i = \left(\frac{v}{s}\right)_i \left(\frac{C}{X_c'}\right) = Y_i \cdot \frac{C}{X_c'}$$

- Check whether:

$$\sum_{i=1}^{n} g_i + L = C$$

Step 6: Determine Yellow and All-red Intervals

- Yellow interval:

$$Y = t_r + \frac{V}{2a + 2gG}$$

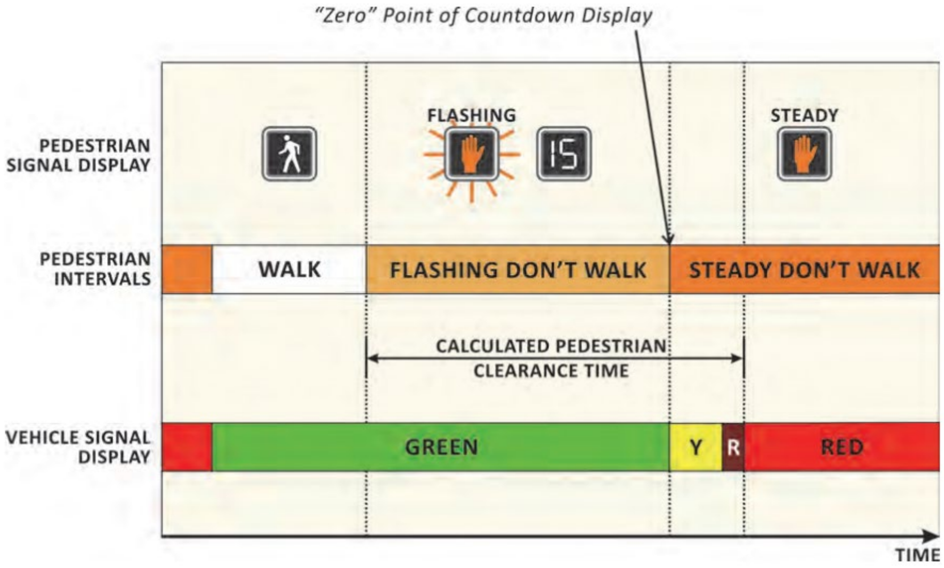
- Red-clearance interval:

$$AR = \frac{w + l}{V}$$

- In practice, yellow interval and red-clearance interval are rounded up to nearest 0.5s
- Total clearance time = $Y + AR$

Step 7: Check Pedestrian Crossing Time

- The minimum pedestrian green time:



$$G_{ped} = 7 + \frac{W}{3.5\ ft/s}$$

W: width of pedestrian crossing

- If minimum pedestrian green time is greater than green time, then green time must be increased (also cycle length)

Step 8: Signal Indication Diagram

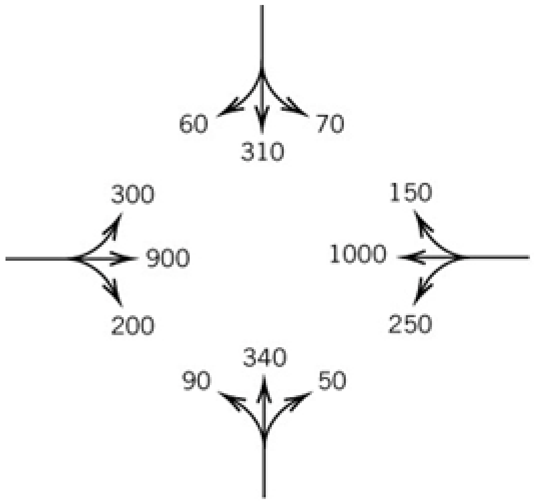
- For each phase i, we have the loss time:

$$G_i + Y_i + AR_i = L_i + g_i$$

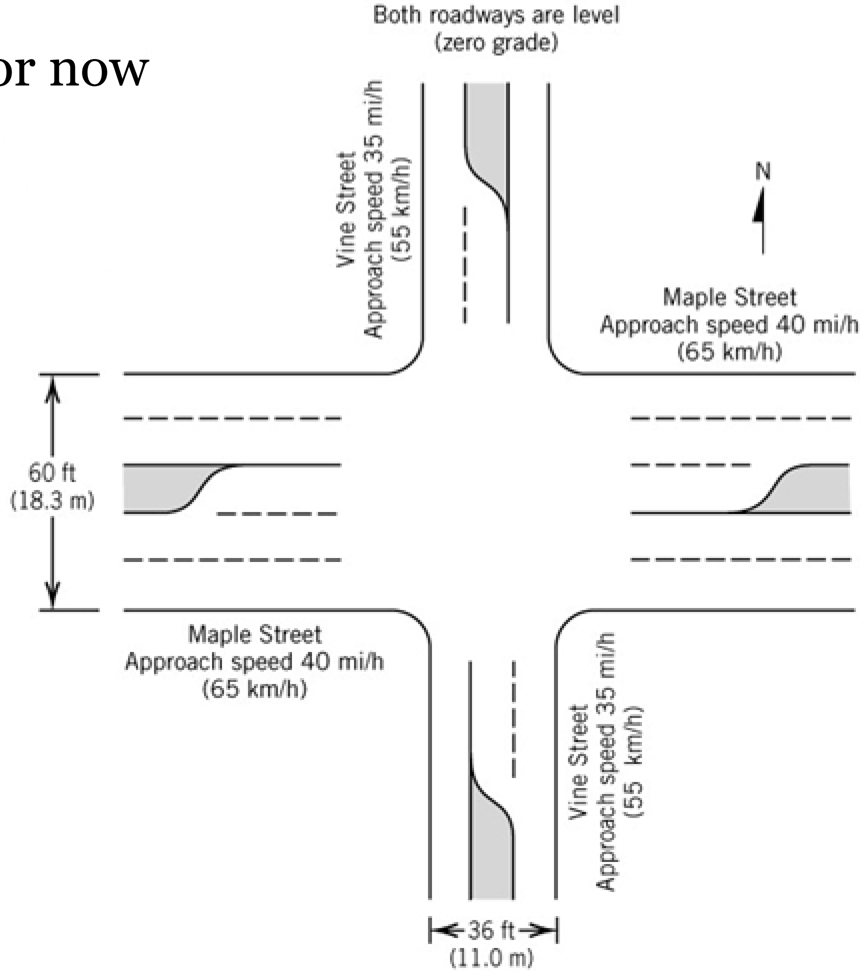
  - G_i: display green
  - Y_i: yellow change interval
  - AR_i: all-red clearance time
  - L_i: loss time
  - g_i: effective green
- We need to get the display green/yellow/red

Signal Timing Example

- Only consider single-ring for now



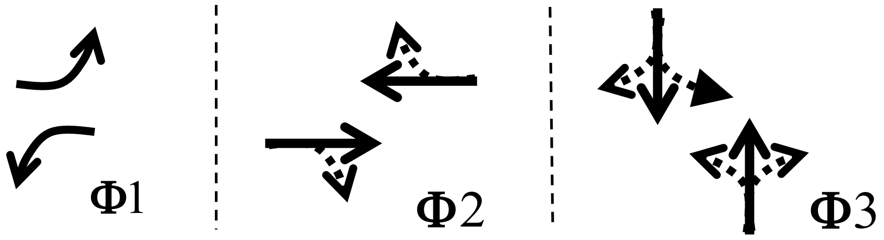
Peak-hour traffic volumes



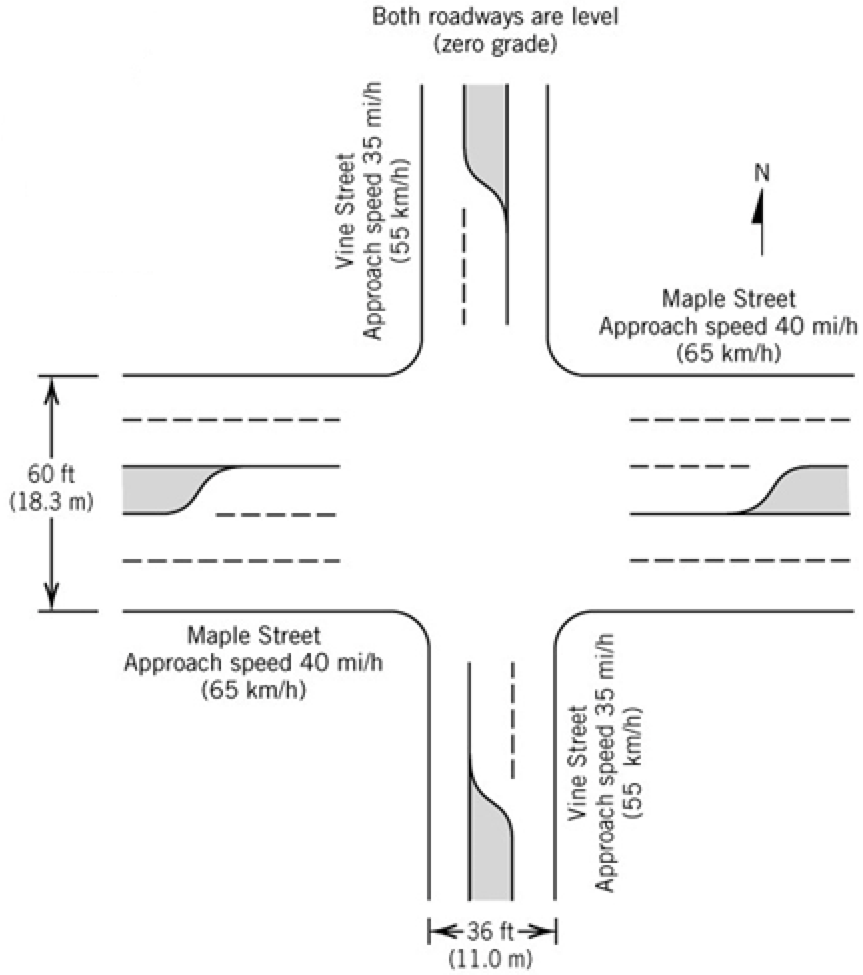
Step 1: Determine Phasing

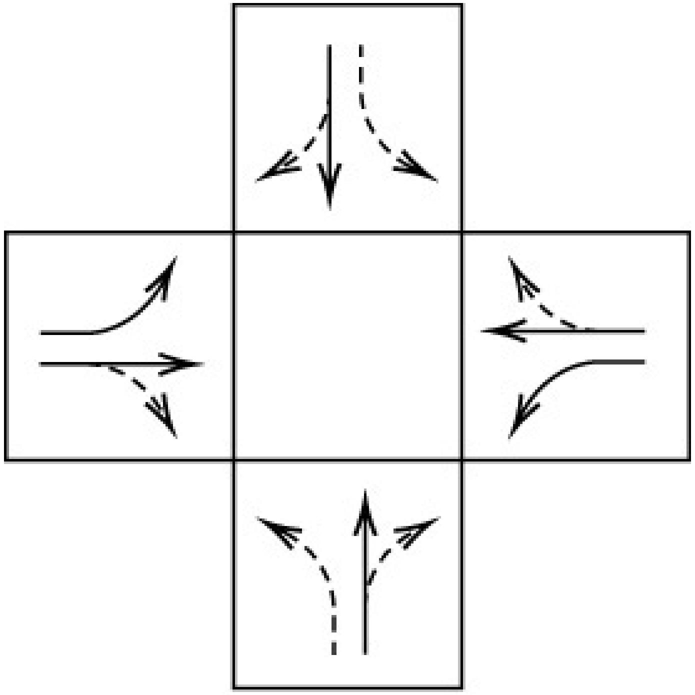
- Eastbound and westbound
  - Eastbound left turn= 300 veh/h >240 veh/h
  - Westbound left turn = 250 veh/h > 240 veh/h
  - Left turn phase needed!
- Northbound and southbound
  - Northbound left turn = 90 veh/h <240 veh/h
  - Southbound left turn = 70 veh/h <240 veh/h
  - Product of through+right and left are smaller than 50000
  - No left turn phase needed!

Three Phases



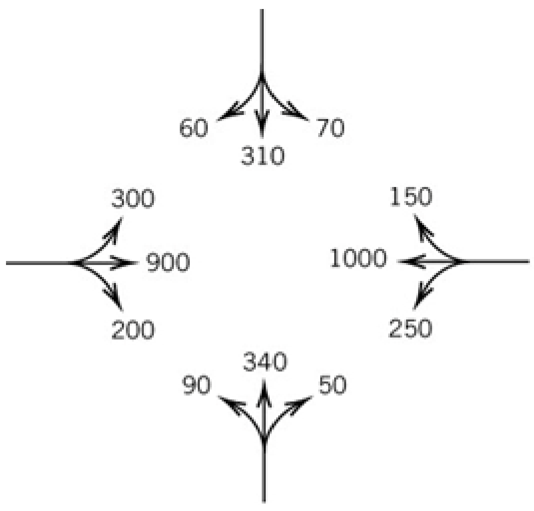
Step 2: Determine Lane Groups



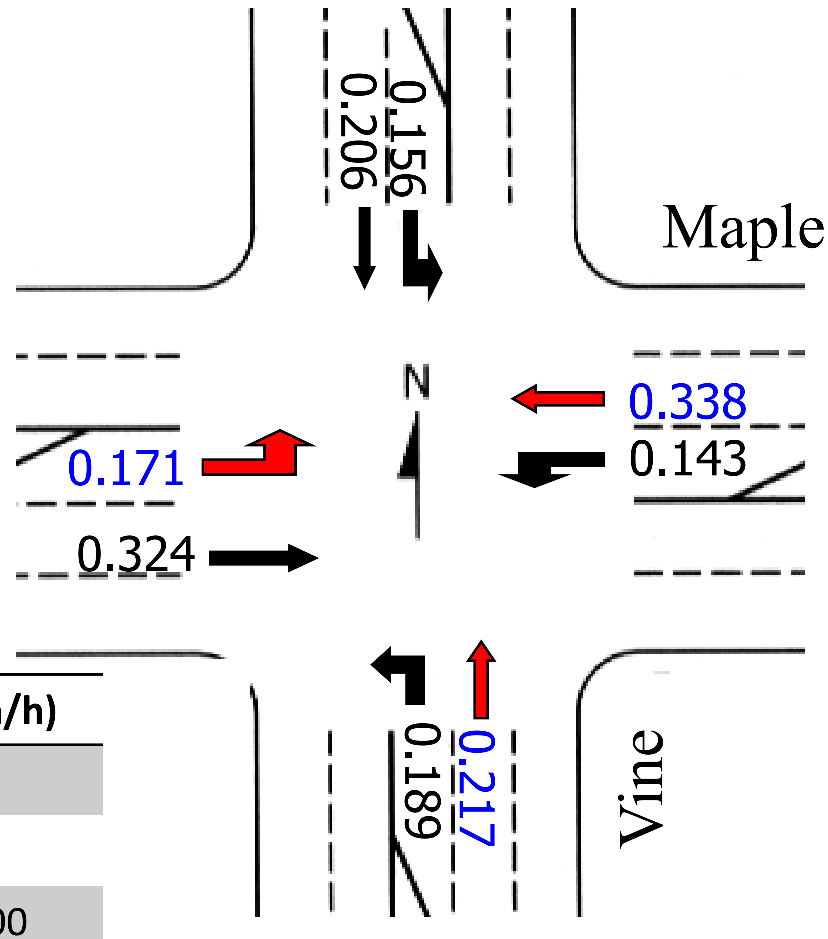


8 lane groups are defined

Step 3: Critical Lane Group v/s Ratio



Peak-hour traffic volumes



Phase 1 (veh/h)	Phase 2 (veh/h)	Phase 3 (veh/h)
EB L: 1750	EB T/R: 3400	SB L: 450
		NB L: 475
WB L: 1750	WB T/R: 3400	SB T/R: 1800
		SB T/R: 1800

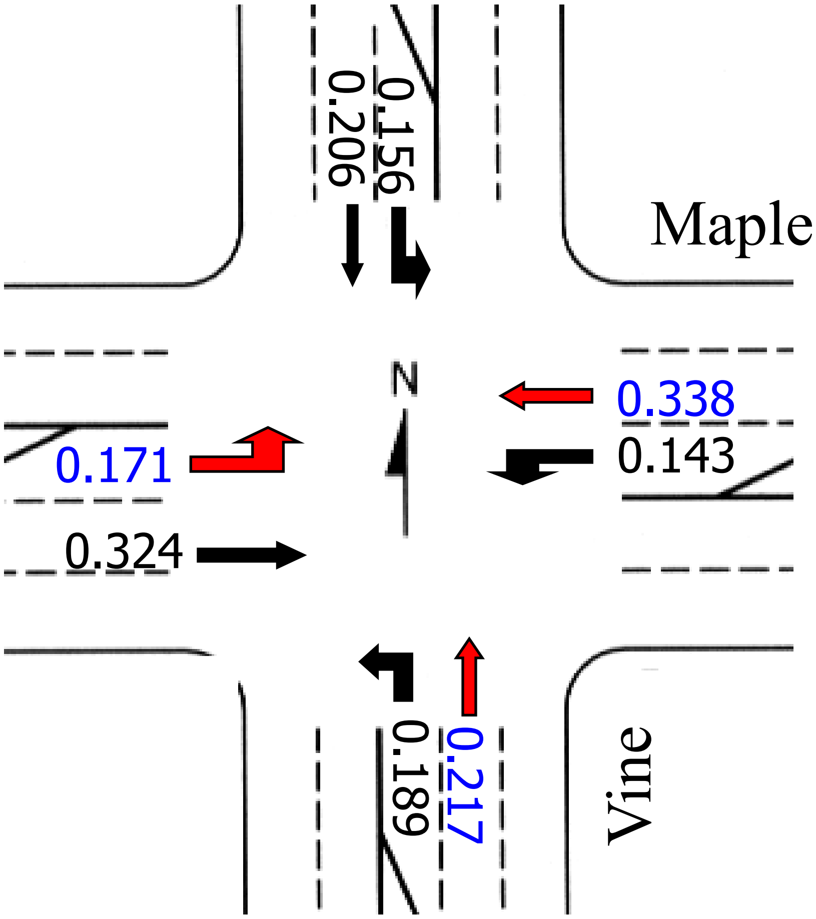
Step 4: Determine Cycle Length

- Assuming lost time is 4 seconds per phase.
- The optimal cycle length is:

$$C_{opt} = \frac{1.5 \times L + 5}{1.0 - Y_c}$$

$$= \frac{1.5 \times 4 \times 3 + 5}{1.0 - (0.171 + 0.338 + 0.217)} = 83.9$$

- Rounded up to 85s



Step 5: Allocate Green Time

- Calculate X_c':

$$X_c' = \frac{Y_c \times C}{C - L} = \frac{0.726 \times 85}{85 - 12} = 0.845$$

$$g_i = (v_i/c_i) * (C/X_c')$$

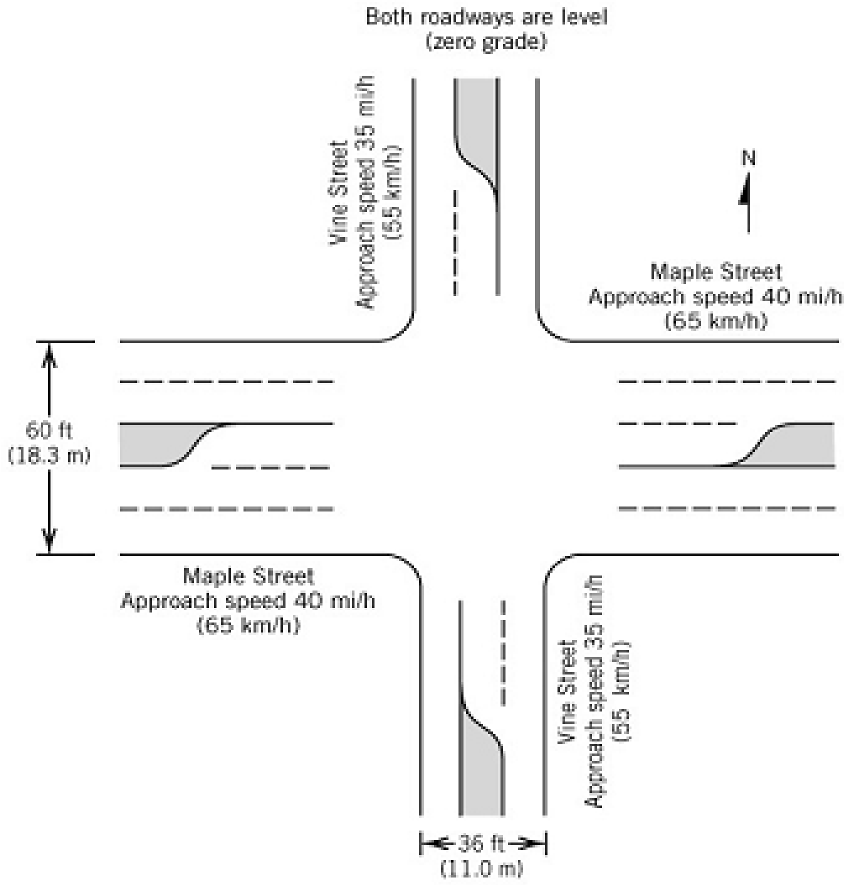
$$g_1 = 0.171 * 85/0.845 = 17.2s -> 17$$

$$g_2 = 0.338 * 85/0.845 = 34s -> 34$$

$$g_3 = 0.217 * 85/0.845 = 21.8s -> 22$$

$$g_1 + g_2 + g_3 = 73 = C - L$$

Step 6: Clearance Time



Maple Street:

$$Y = \mathrm{T} + \left(\frac{v}{2a + 2Gg}\right)$$

$$Y = 1.0 + \left(\frac{40 * 5280/3600}{2 * 10}\right)$$

$$\boxed{Y = 3.9 => 4.0\text{sec}}$$

$$AR = \frac{w + l}{v}$$

$$\boxed{AR = \frac{36 + 20}{40 * 5280/3600} = 1.0\text{sec}}$$

Step 6 Clearance Time

- Vine Street

$$Y = \mathrm{T} + \left(\frac{v}{2a + 2Gg}\right)$$

$$AR = \frac{w + l}{v}$$

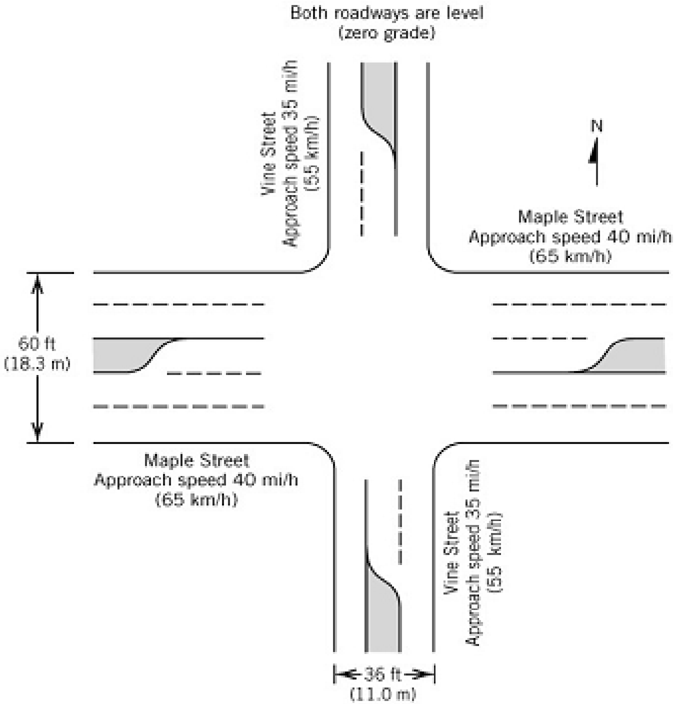
$$Y = 1.0 + \left(\frac{35 * 5280/3600}{2(10)}\right)$$

$$AR = \frac{60 + 20}{35 * 5280/3600}$$

Y = 3.6 => **Y = 4.0 sec**

AR = 1.6 => **AR = 2.0 sec**

Step 7: Check Pedestrian Time



Maple:

$$G_{ped} = 7 + \frac{60}{3.5} = 24.2\ s$$

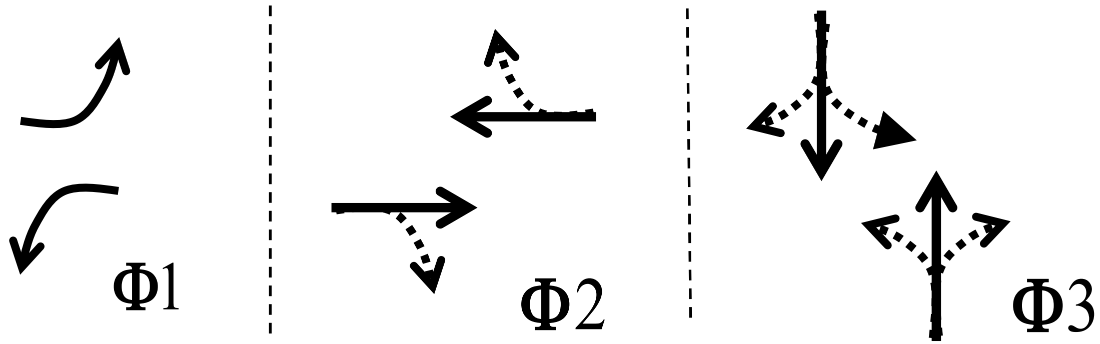
Phase 2: 34 sec of Green is OK

Vine

$$G_{ped} = 7 + \frac{36}{3.5} = 17.2\ s$$

Phase 3: 22 sec of Green is OK

Step 8: Summarize Signal Timing



Green Display = Effective Green + lost time – Y - AR

Φ1: G = 17 + 4 – 4 – 1 =16, Y=4, AR=1
Φ2: G = 34 + 4 – 4 – 1 =33, Y=4, AR=1
Φ3: G = 22+ 4 – 4 – 2 =20, Y=4, AR=2
C = 85 s

Level of Service Analysis

- Analysis Procedure (assuming signal timing plan has already been determined)
  - Determine the capacities (service) and volumes (arrivals)
  - Calculate delay
  - Determine Level of Service (LOS)
- LOS and Delays

Highway Capacity Manual Level of Service Criteria for Signalized Intersections

Level of service	Average control delay (seconds/vehicle)	Description
A	≤10	Free flow
B	>10–20	Stable flow (slight delay)
C	>20–35	Stable flow (acceptable delays)
D	>35–55	Approaching unstable flow (tolerable delay)
E	>55–80	Unstable flow (intolerable delay)
F	>80	Forced flow (congested and queues fail to clear)

Level of Service and Delays

Highway Capacity Manual Level of Service Criteria for Signalized Intersections

Level of service	Average control delay (seconds/vehicle)	Description
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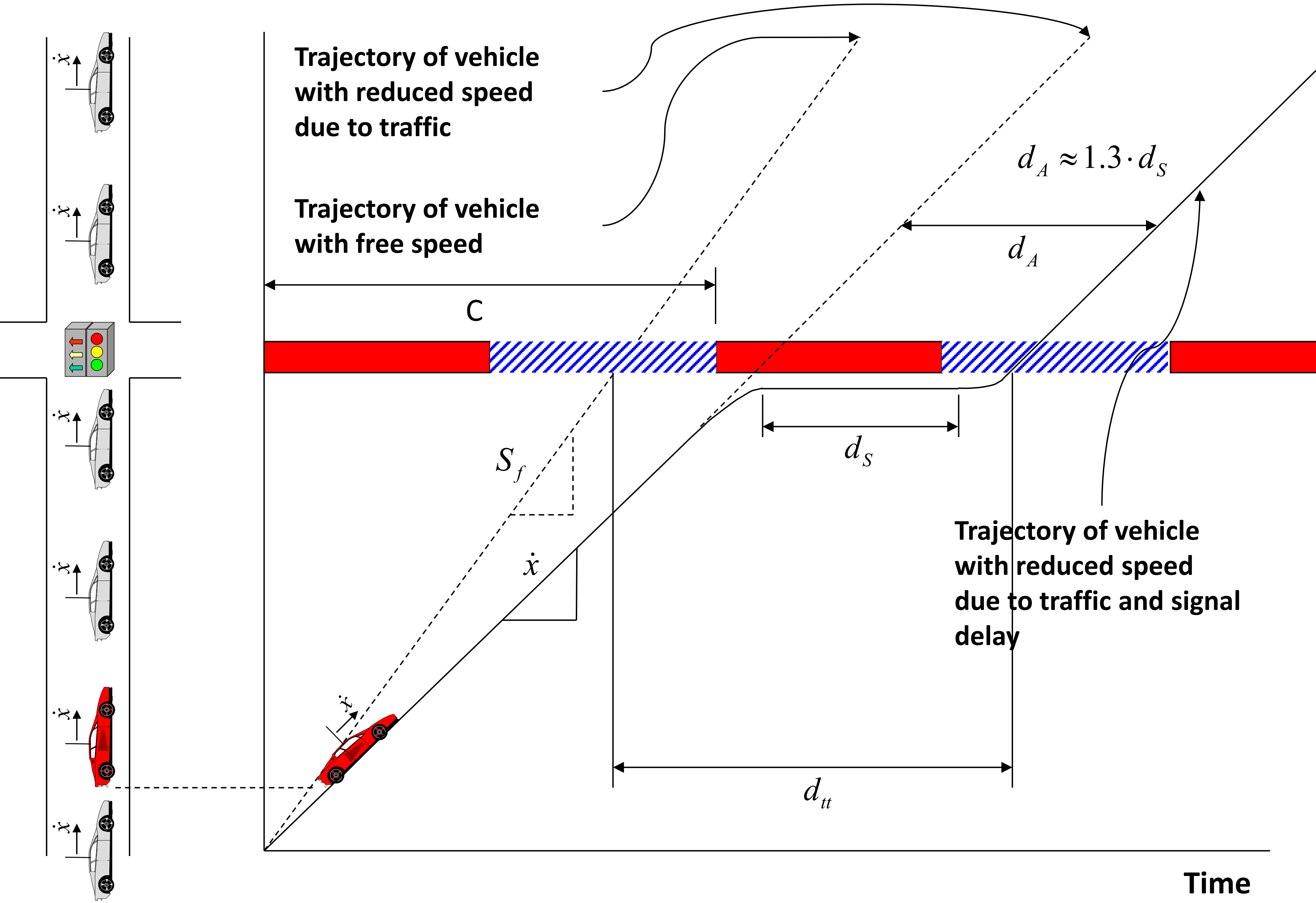
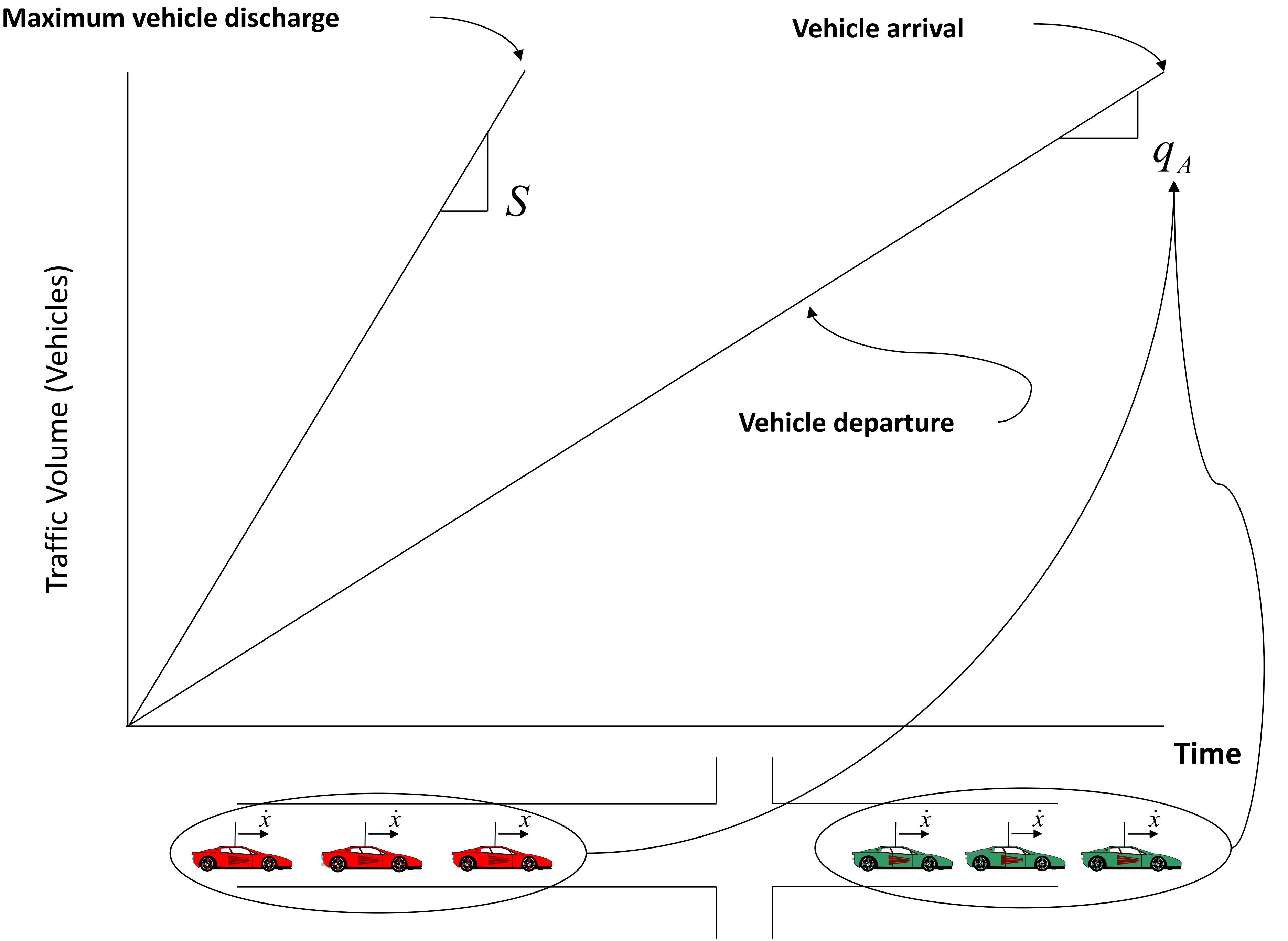
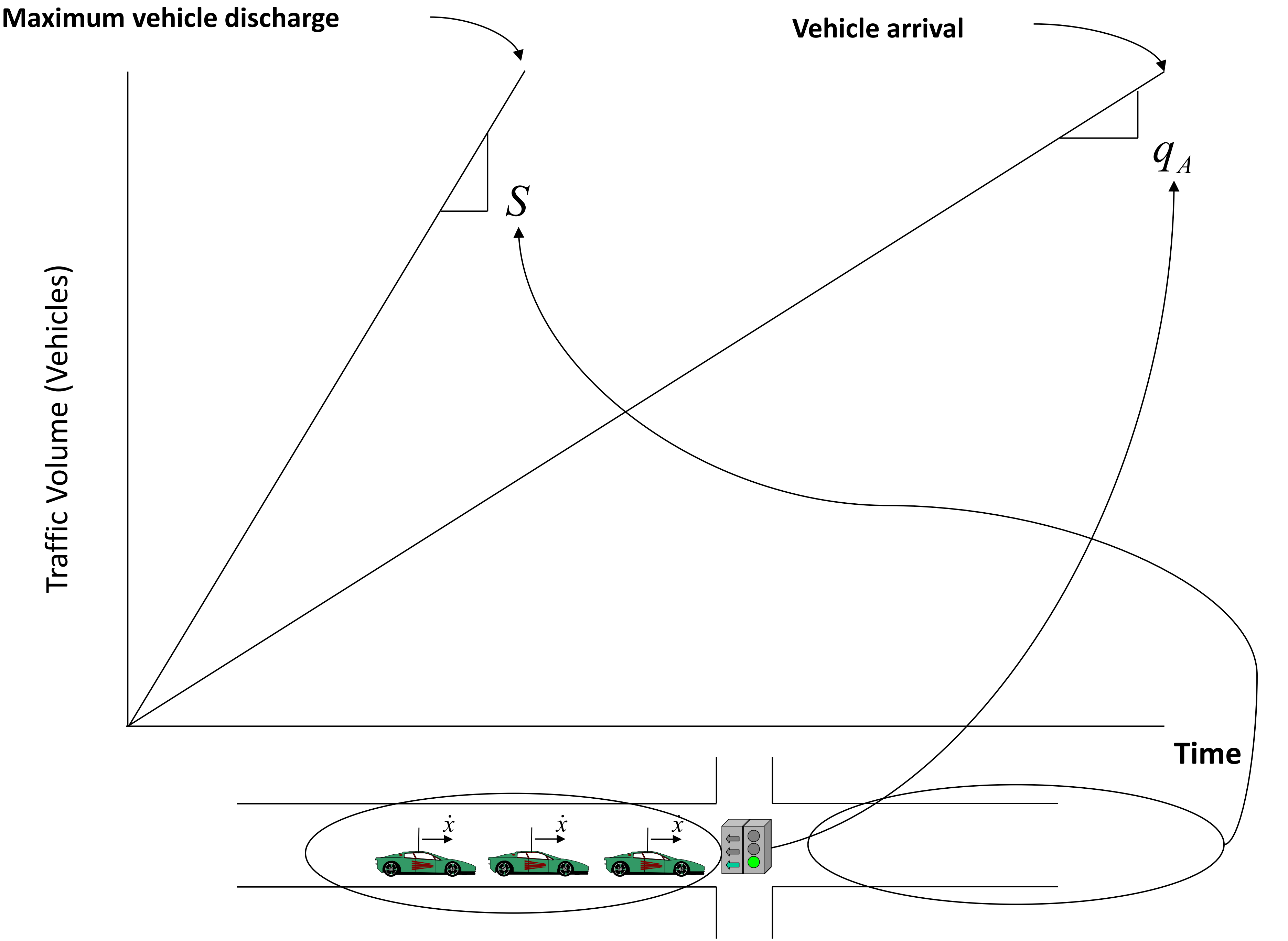


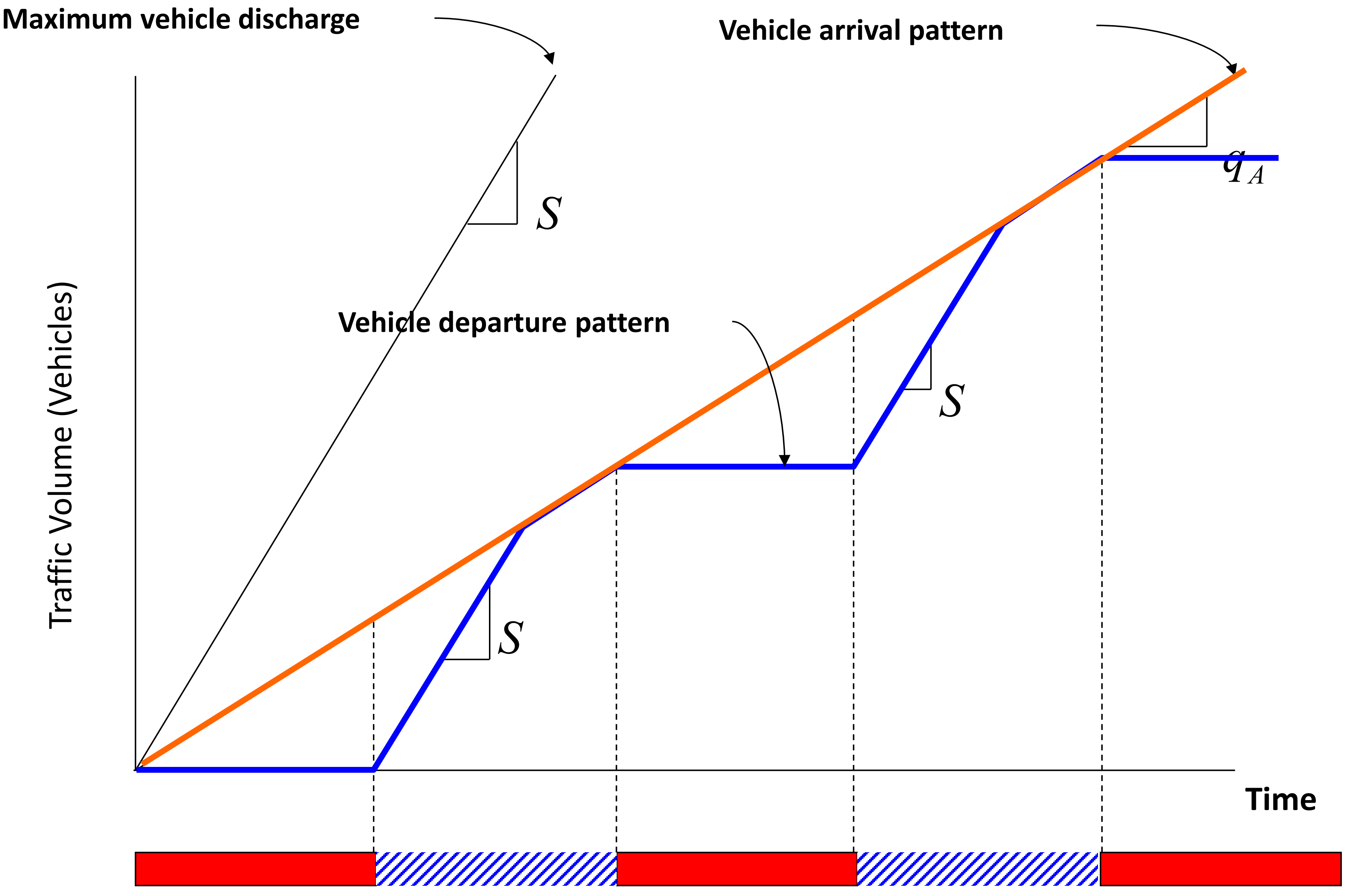
Trajectory of vehicle with reduced speed due to traffic
Trajectory of vehicle with free speed
$d_A \approx 1.3 \cdot d_S$
d_A
C
d_S
S_f
$\dot{x}$
Trajectory of vehicle with reduced speed due to traffic and signal delay
d_{tt}
Time



Maximum vehicle discharge
Vehicle arrival
S
q_A
Traffic Volume (Vehicles)
Vehicle departure
Time
$\dot{x}$
$\dot{x}$
$\dot{x}$
$\dot{x}$
$\dot{x}$
$\dot{x}$



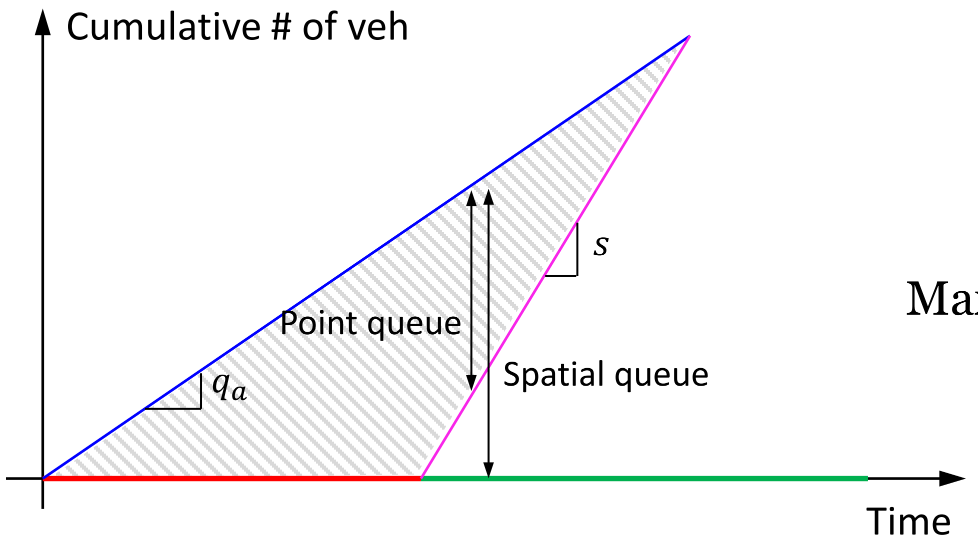
Maximum vehicle discharge
Vehicle arrival
S
q_A
Traffic Volume (Vehicles)
Time
$\dot{x}$
$\dot{x}$
$\dot{x}$



Maximum vehicle discharge
Vehicle arrival pattern
S
q_A
Vehicle departure pattern
S
S
Traffic Volume (Vehicles)
Time

Point Queue & Spatial Queue

- Queueing diagram



Maximum queue?

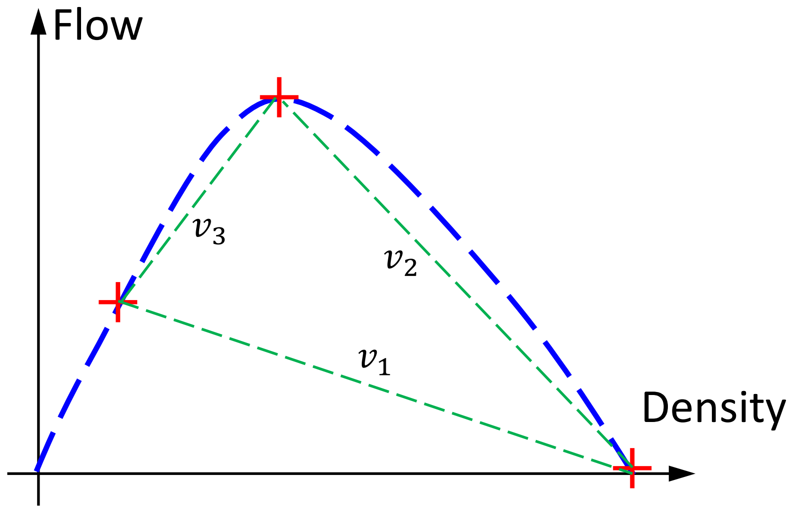
  - Point queue: number of waiting (stopping) vehicles
  - Spatial queue: location of the end of queue
  - Total delay: shadow area, unit: veh*sec

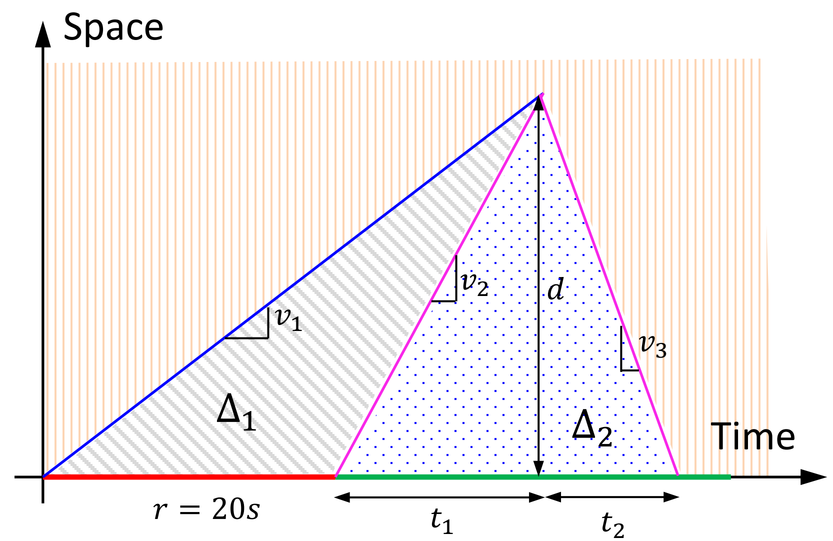
Readings

- Signal Timing Manual (2^{nd} Edition): Chapter 5

Time-Space Diagram

- Time-space diagram: different shockwave areas

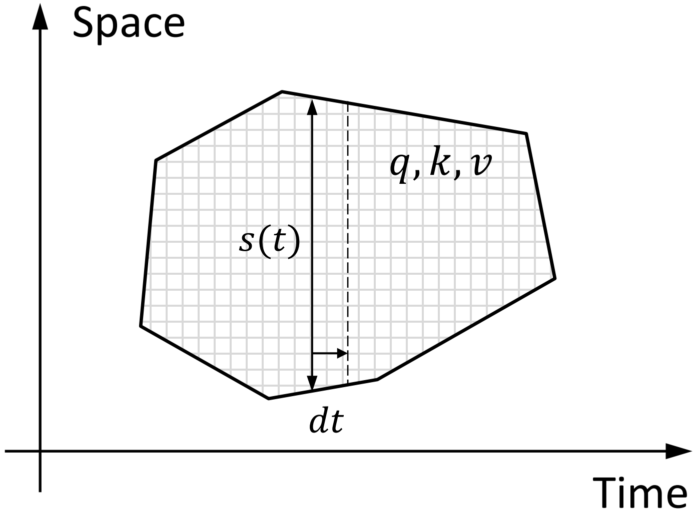




- TS diagram & queueing diagram are equivalent with the triangle fundamental diagram (only free flow & jam, or stop and go)
- How we calculate the total delay in TS diagram?

Total Travel Time in the Time-Space Diagram

- Obtain the travel time given the time-space diagram



- Number of vehicle in dashed area at time t

$$n(t) = s(t) \cdot k$$

- Total delay from t to $t + dt$

$$n(t) \cdot dt$$

– What is the total travel time when the vehicle passing by the dashed area?

$$TTT = \int n(t)dt = k \int s(t)dt = k \cdot S_0$$

Check the unit!

– S_0: area in the time-space diagram