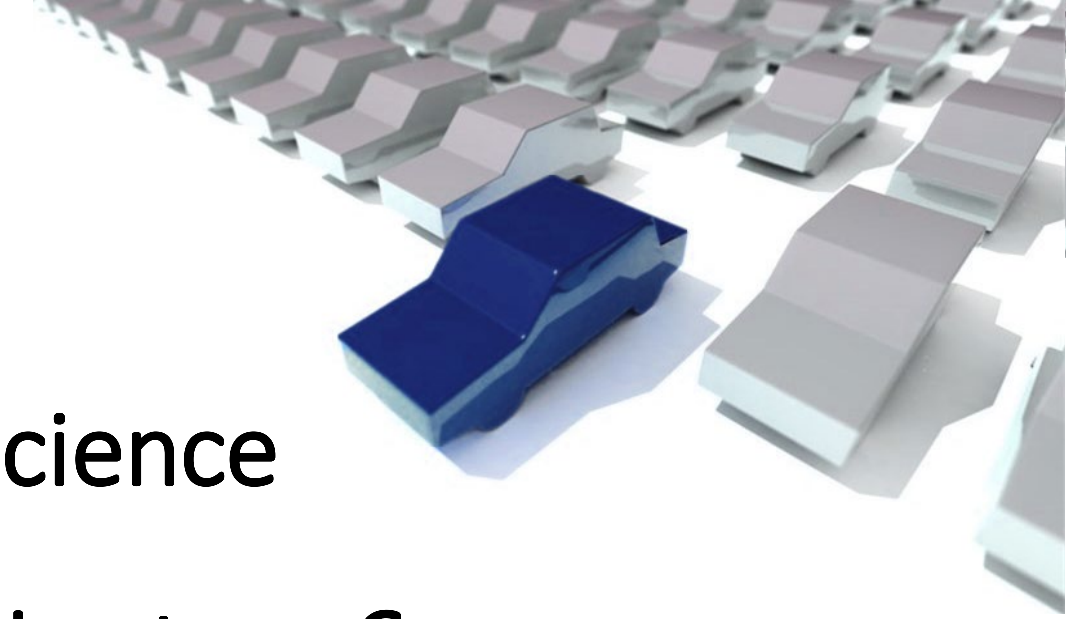

CEE 551 Traffic Science

Traffic Flow Theory Lecture 6

Probabilistic time-space model near signalized intersections

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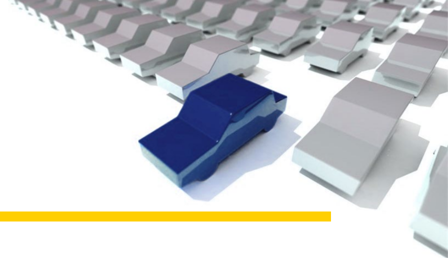
Email: xingminw@umich.edu



Outline

- ☐ Introduction of the Newellian coordinates
- ☐ Point-queue encoding of vehicle trajectories
- ☐ Probabilistic time-space diagram
- ☐ Parameter calibration and traffic signal retiming in the field

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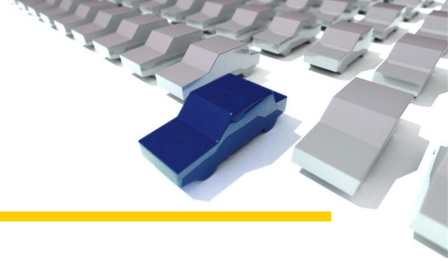
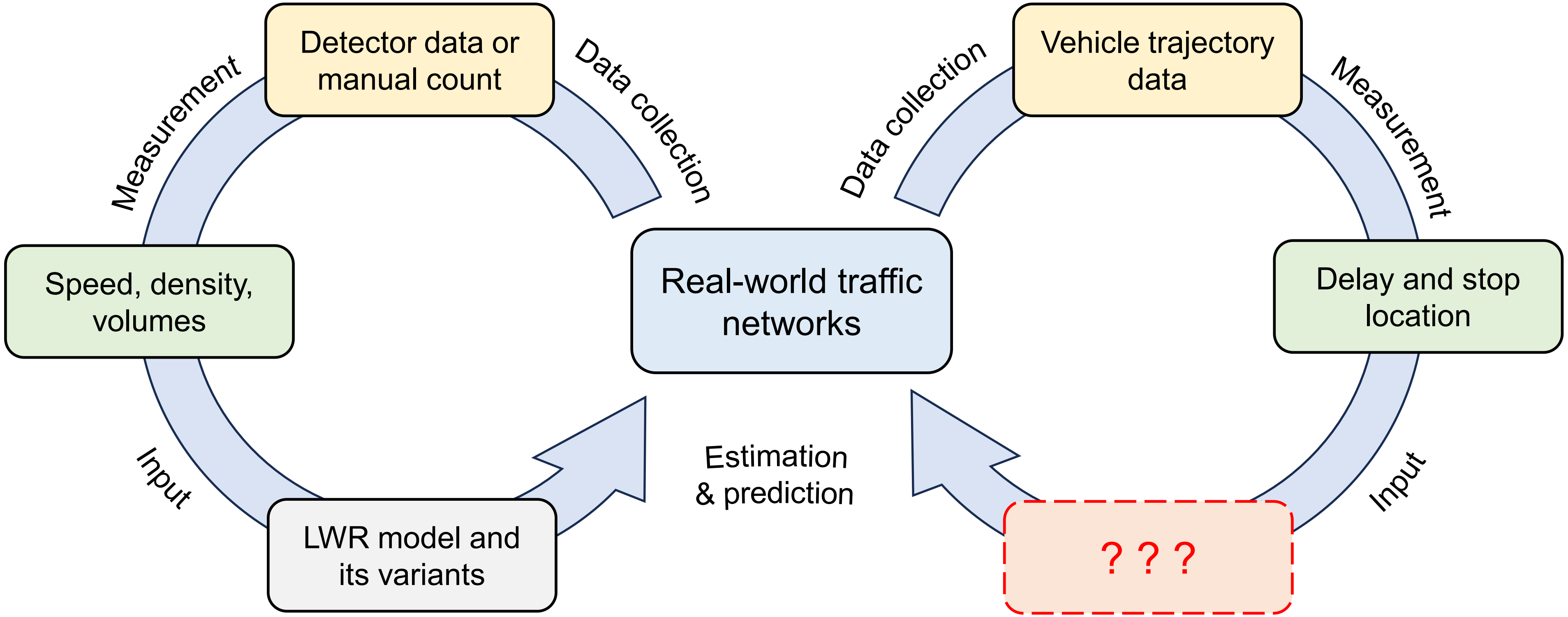
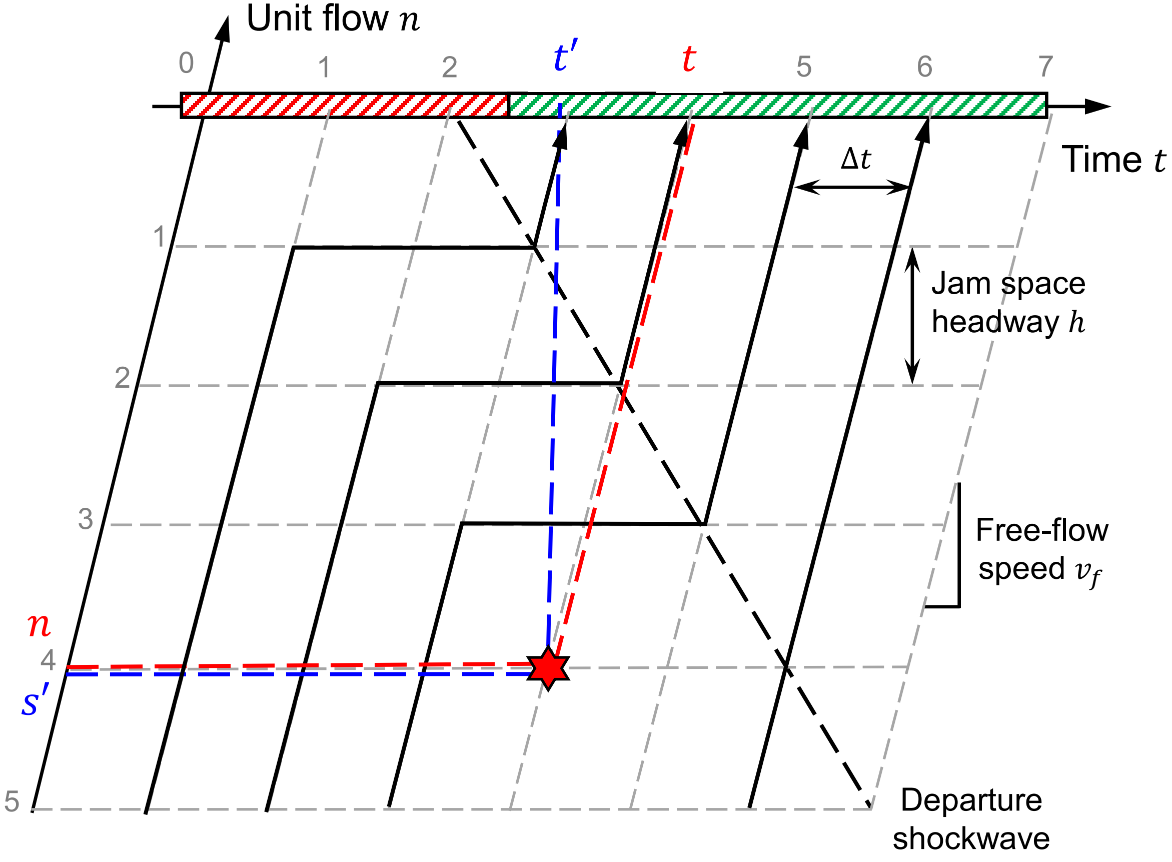


Traffic flow model for low penetration rate vehicle trajectory data



Newellian coordinates

☐ Establishment of Newellian coordinates



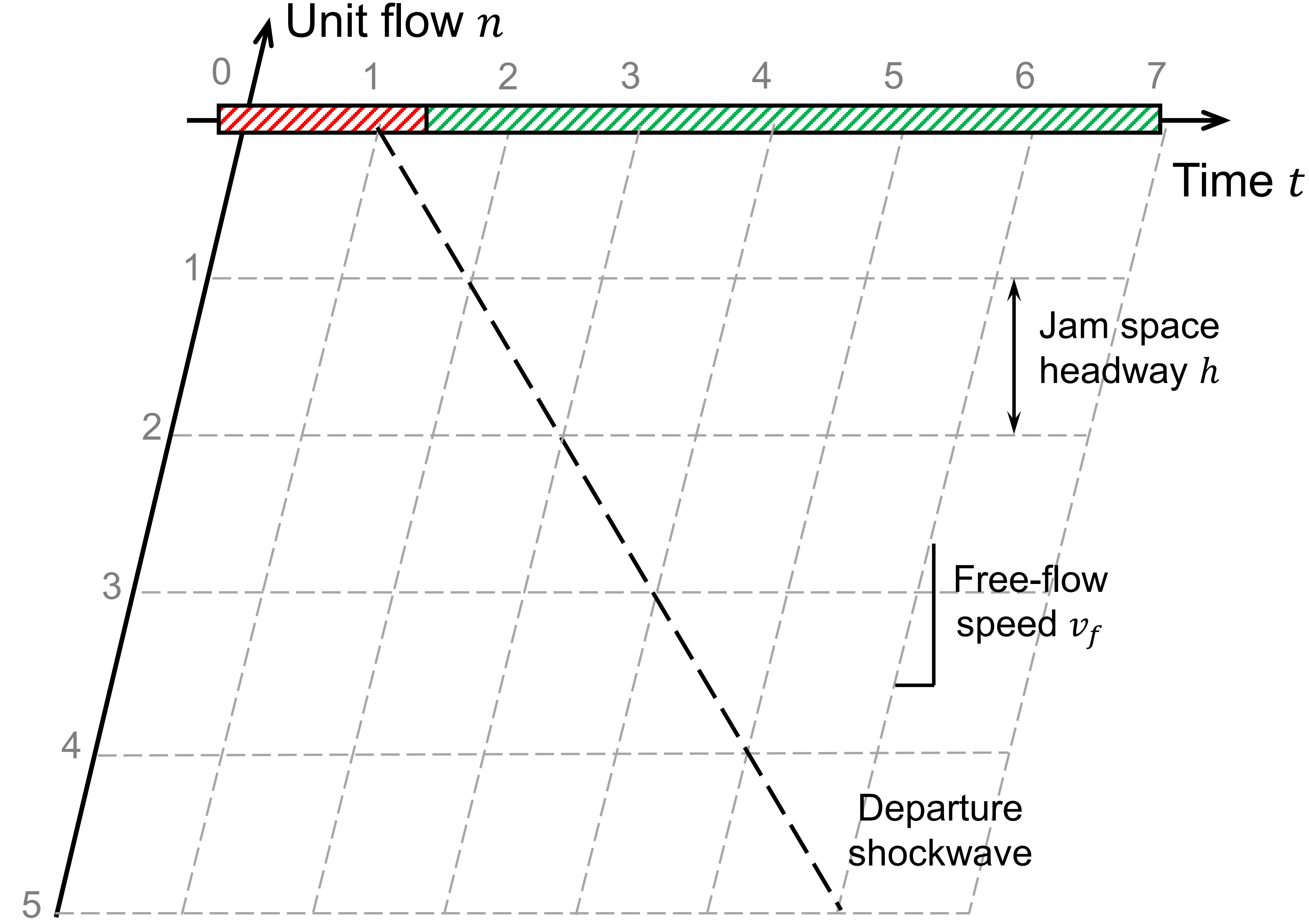
Vehicles only travel on the grid of the Newellian coordinates

- ***Assumption***: All vehicle trajectories follow a uniform deterministic Newell's car-following model. Vehicles only have stop state & free flow state
- ***Discrete approximation***: traffic flow comes in binary (0 or Δu) for each time

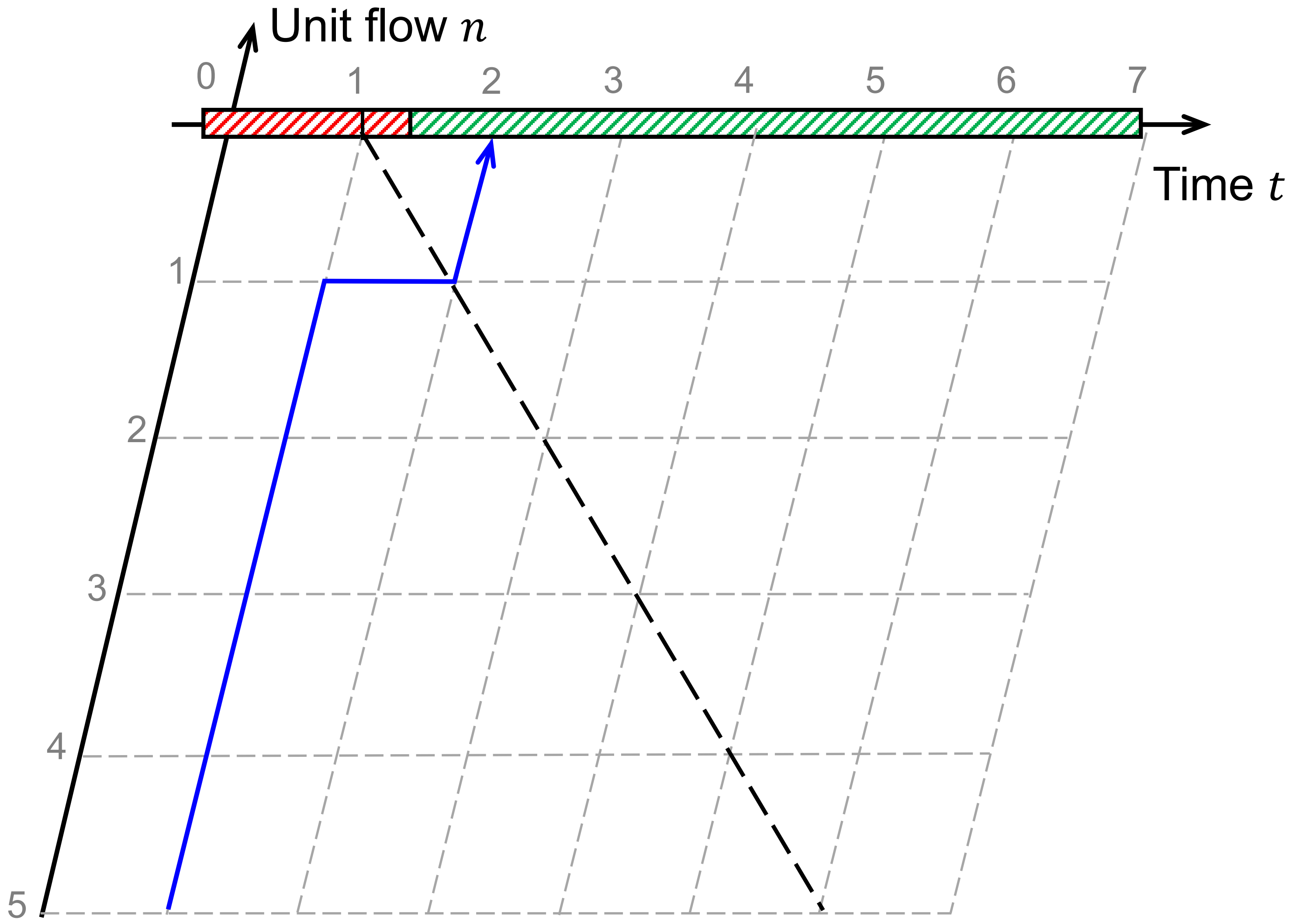
$$\Delta u = q^m z \Delta t \qquad h = \frac{\Delta u \cdot h_0}{z} = q^m h_0 \Delta t$$

Notation	Meaning
Δu	Unit traffic flow
q^m	Saturation flow rate
z	Number of lanes
Δt	Time interval
h	Jam space headway (per Δu)
h_0	Jam space headway (per vehicle)

Point-Queue Under Newellian Coordinates

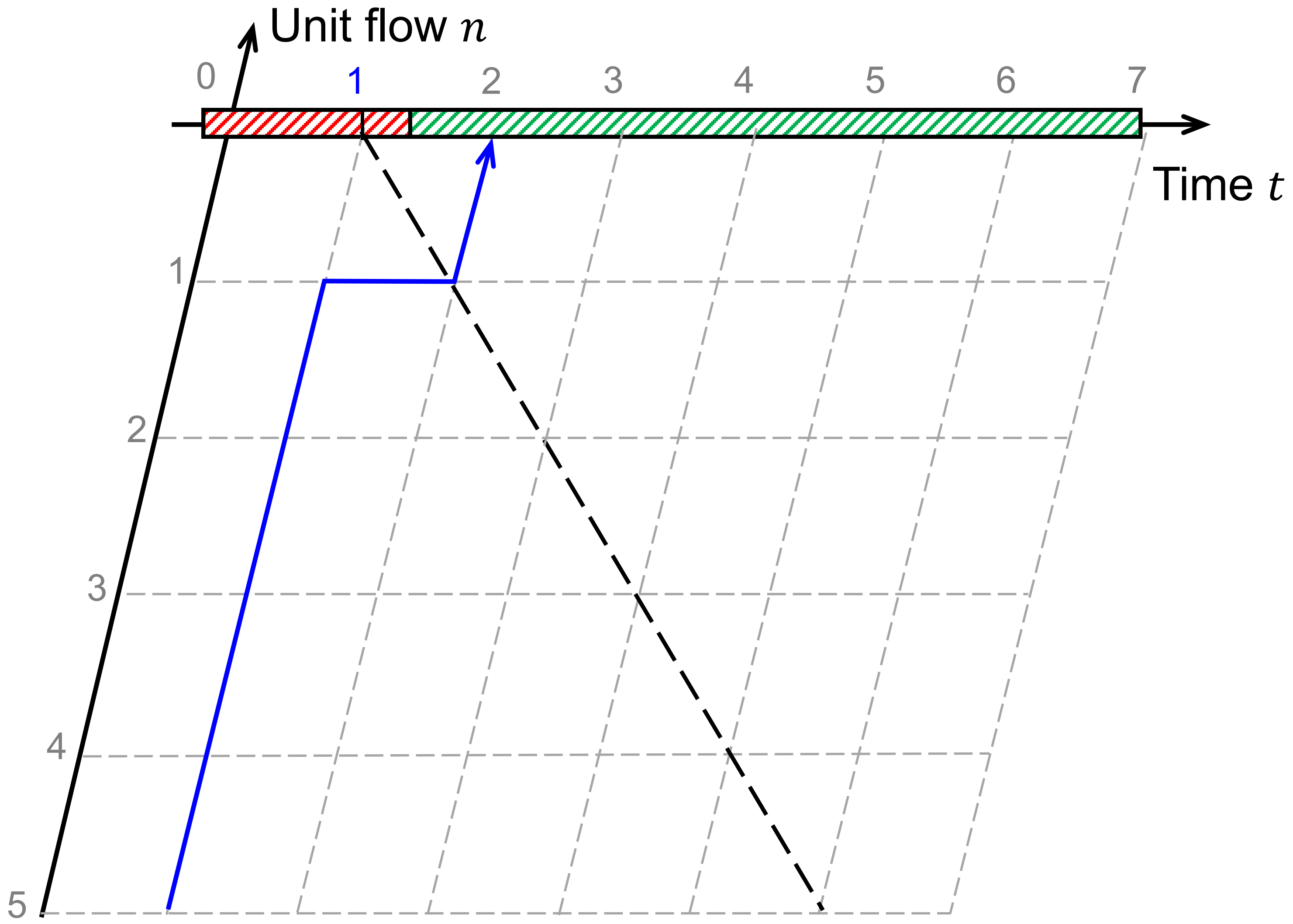


Point-Queue Under Newellian Coordinates



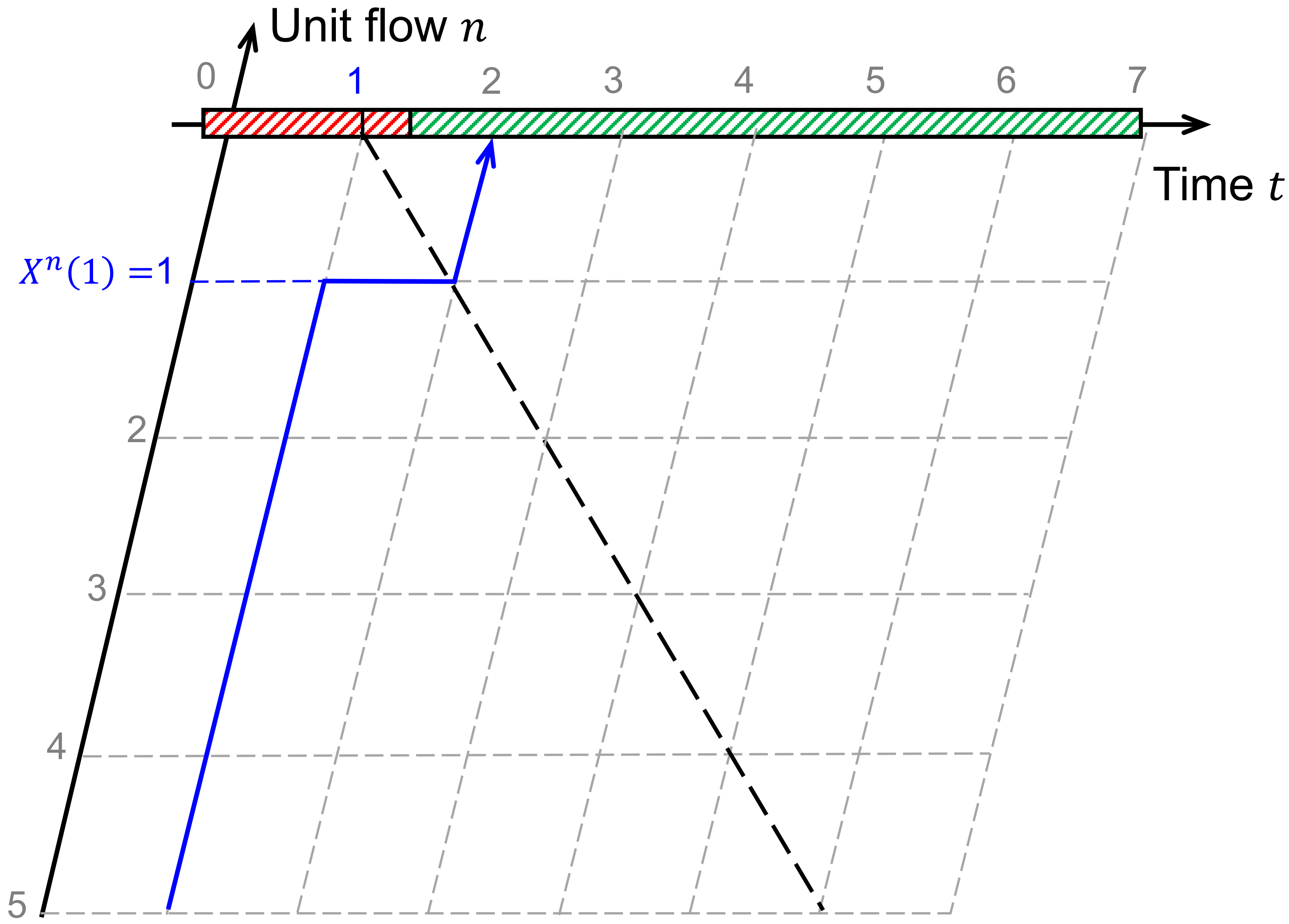
Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1				
2				
3				
4				
5				
6				
7				

Point-Queue Under Newellian Coordinates



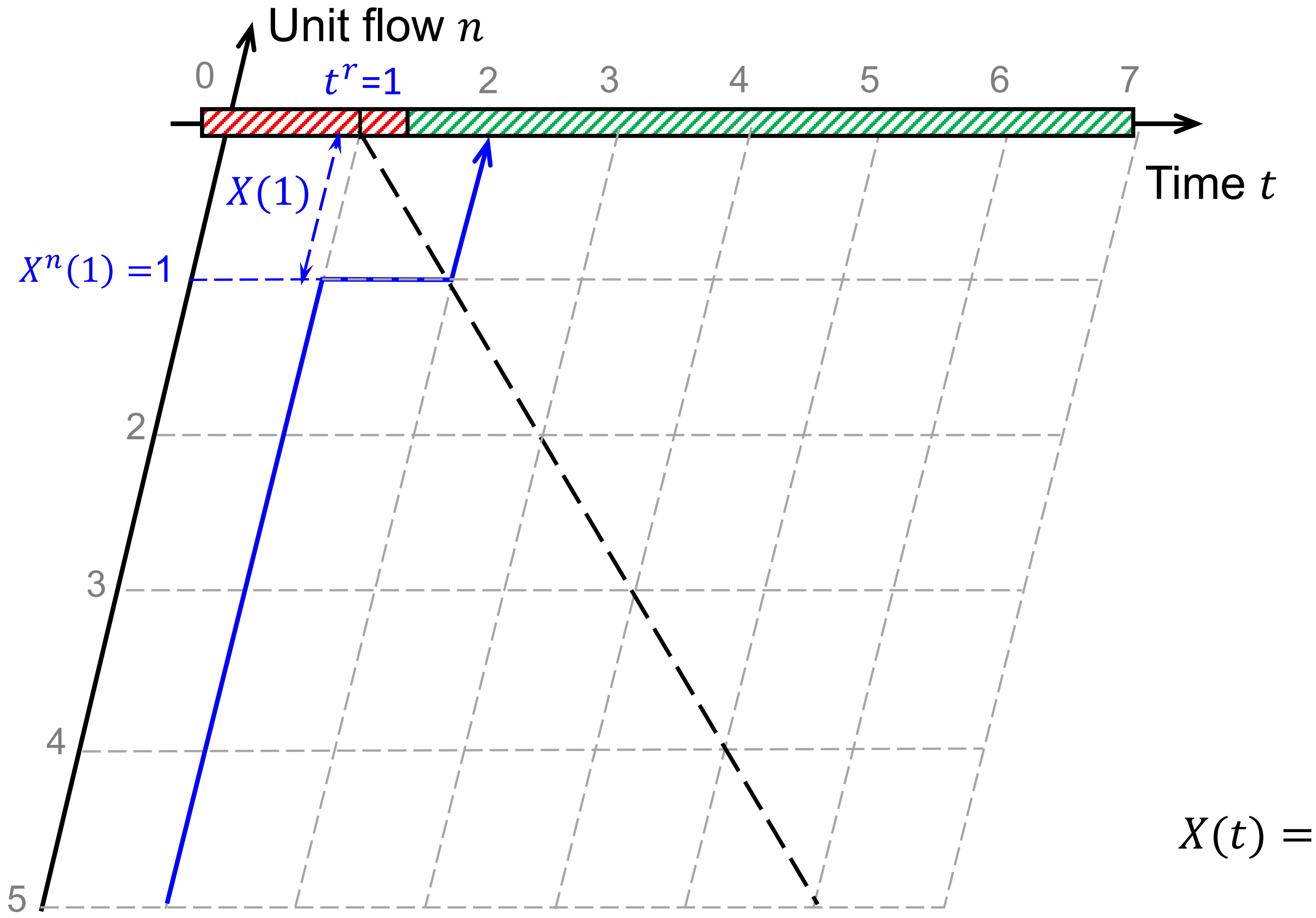
Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1	1			
2				
3				
4				
5				
6				
7				

Point-Queue Under Newellian Coordinates



Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1	1	1		
2				
3				
4				
5				
6				
7				

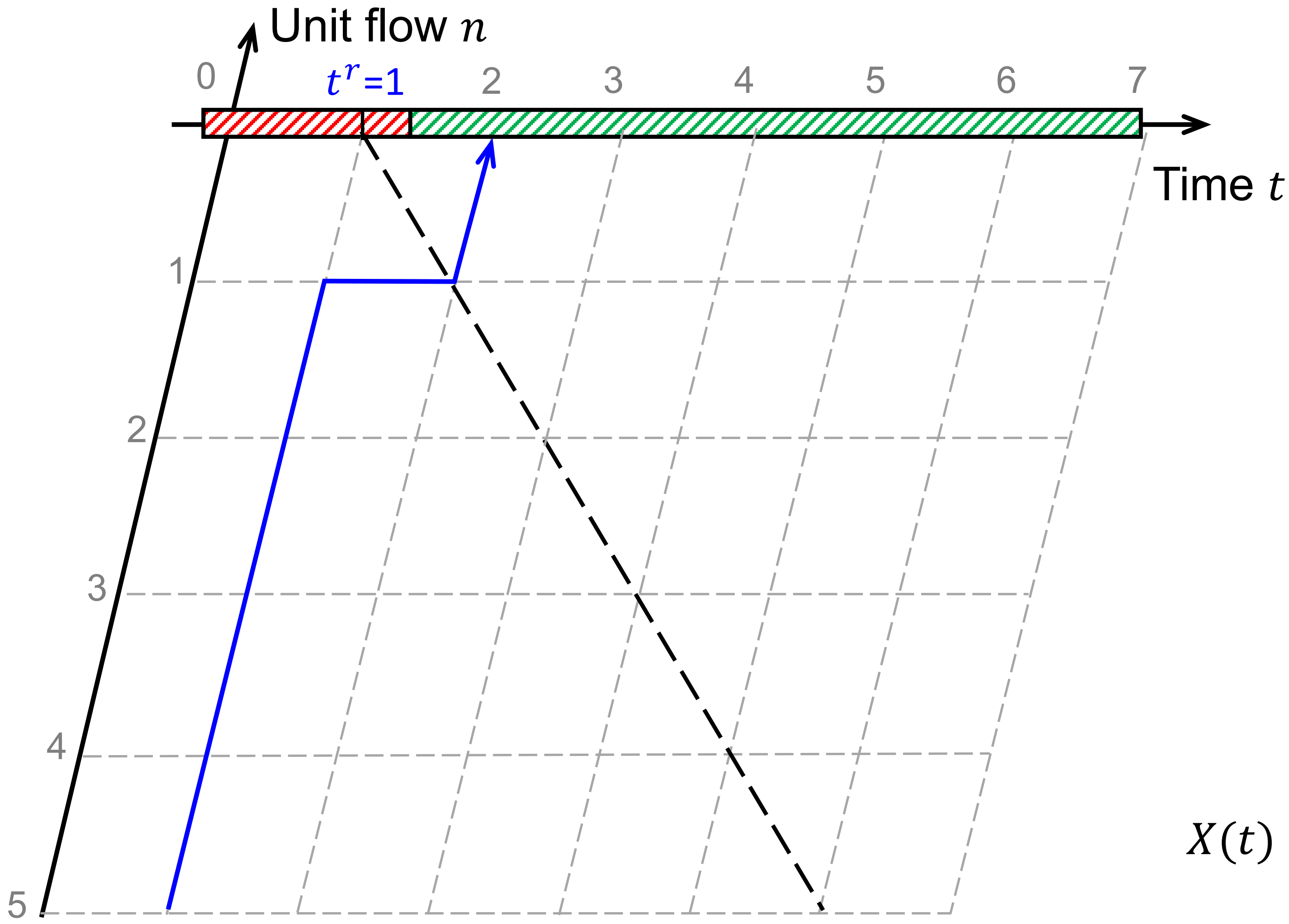
Point-Queue Under Newellian Coordinates



Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1	1	1	1	
2				
3				
4				
5				
6				
7				

$$X(t) = \begin{cases} X^n(t) - \max\{0, t - t^r\}, & X^n(t) > 0 \\ 0, & X^n(t) = 0 \end{cases}$$

Point-Queue Under Newellian Coordinates



Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1	1	1	1	0
2				1
3				
4				
5				
6				
7				

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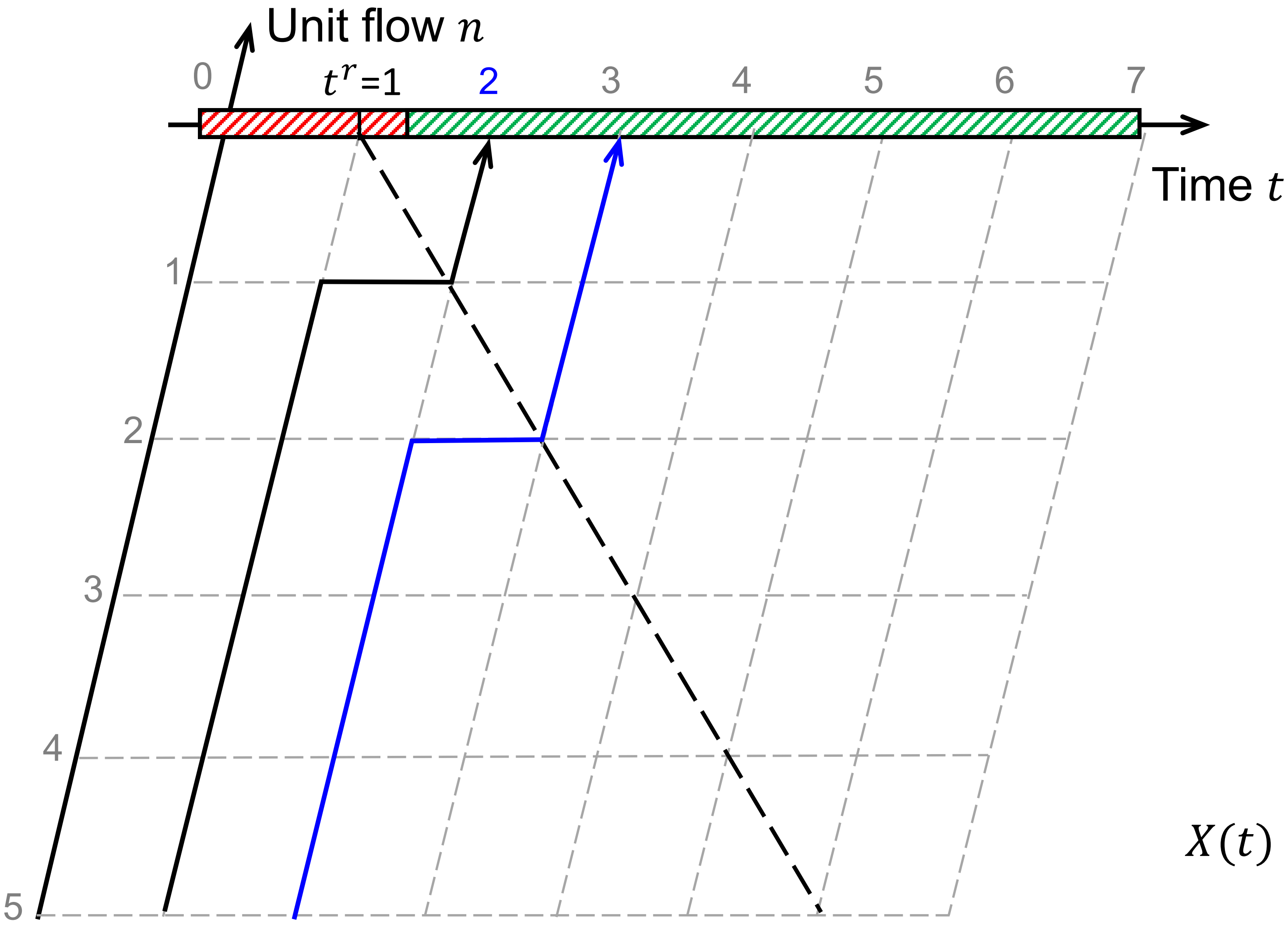
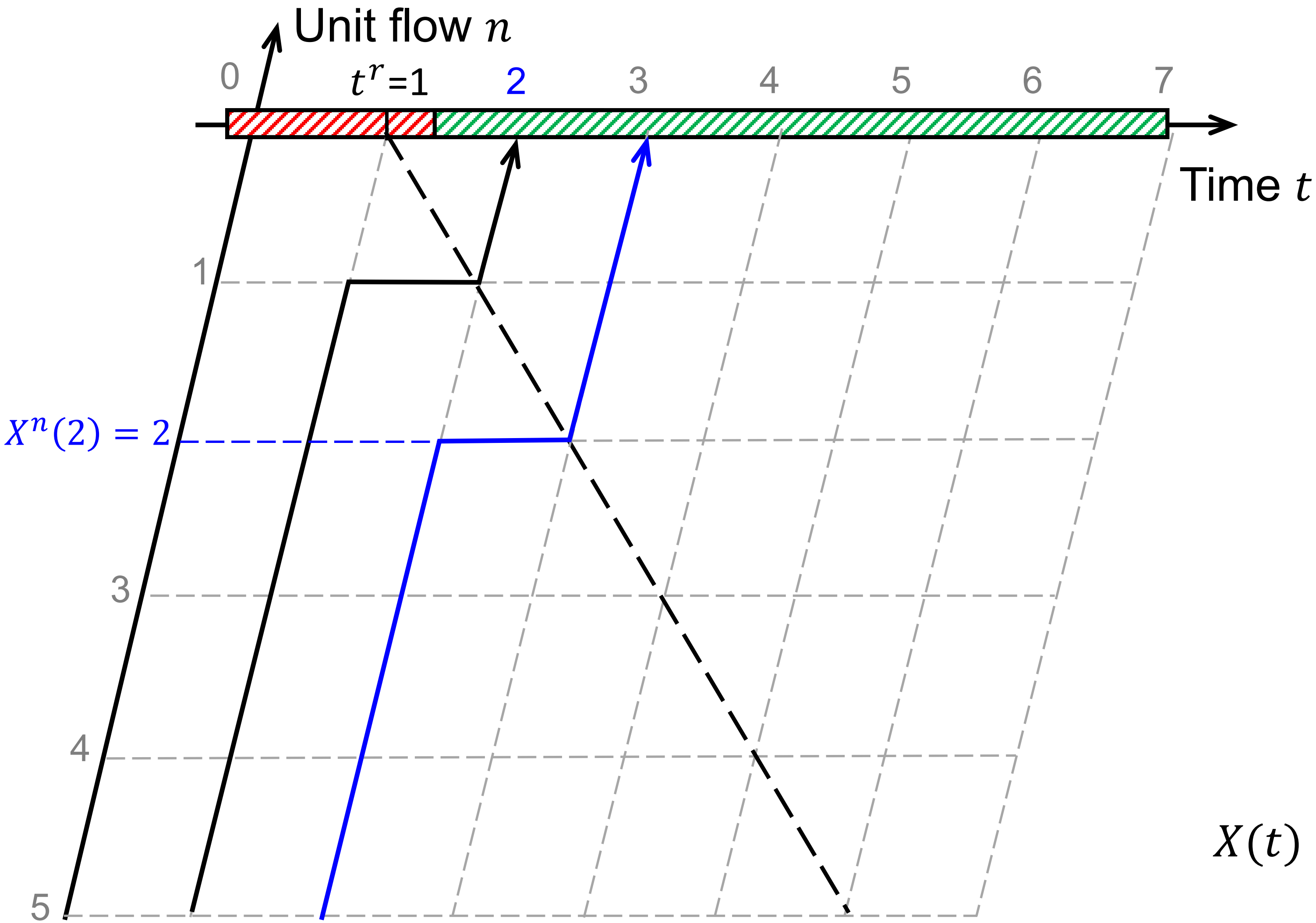
Point-Queue Under Newellian Coordinates



Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1	1	1	1	0
2	1			1
3				
4				
5				
6				
7				

$$X(t) = \begin{cases} X^n(t) - \max\{0, t - t^r\}, & X^n(t) > 0 \\ 0, & X^n(t) = 0 \end{cases}$$

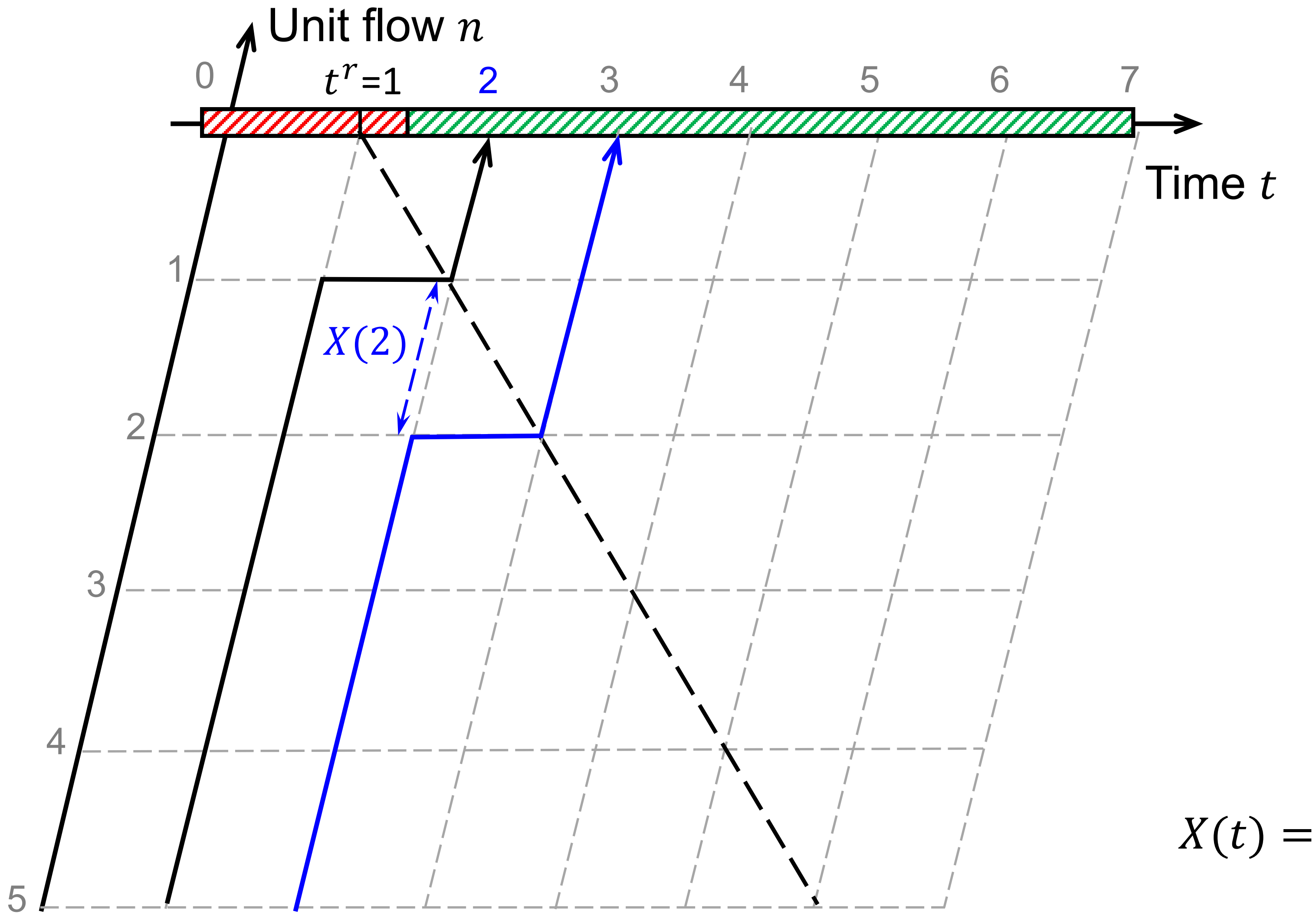
Point-Queue Under Newellian Coordinates



Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1	1	1	1	0
2	1	2		1
3				
4				
5				
6				
7				

$$X(t) = \begin{cases} X^n(t) - \max\{0, t - t^r\}, & X^n(t) > 0 \\ 0, & X^n(t) = 0 \end{cases}$$

Point-Queue Under Newellian Coordinates



Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1	1	1	1	0
2	1	2	1	1
3				
4				
5				
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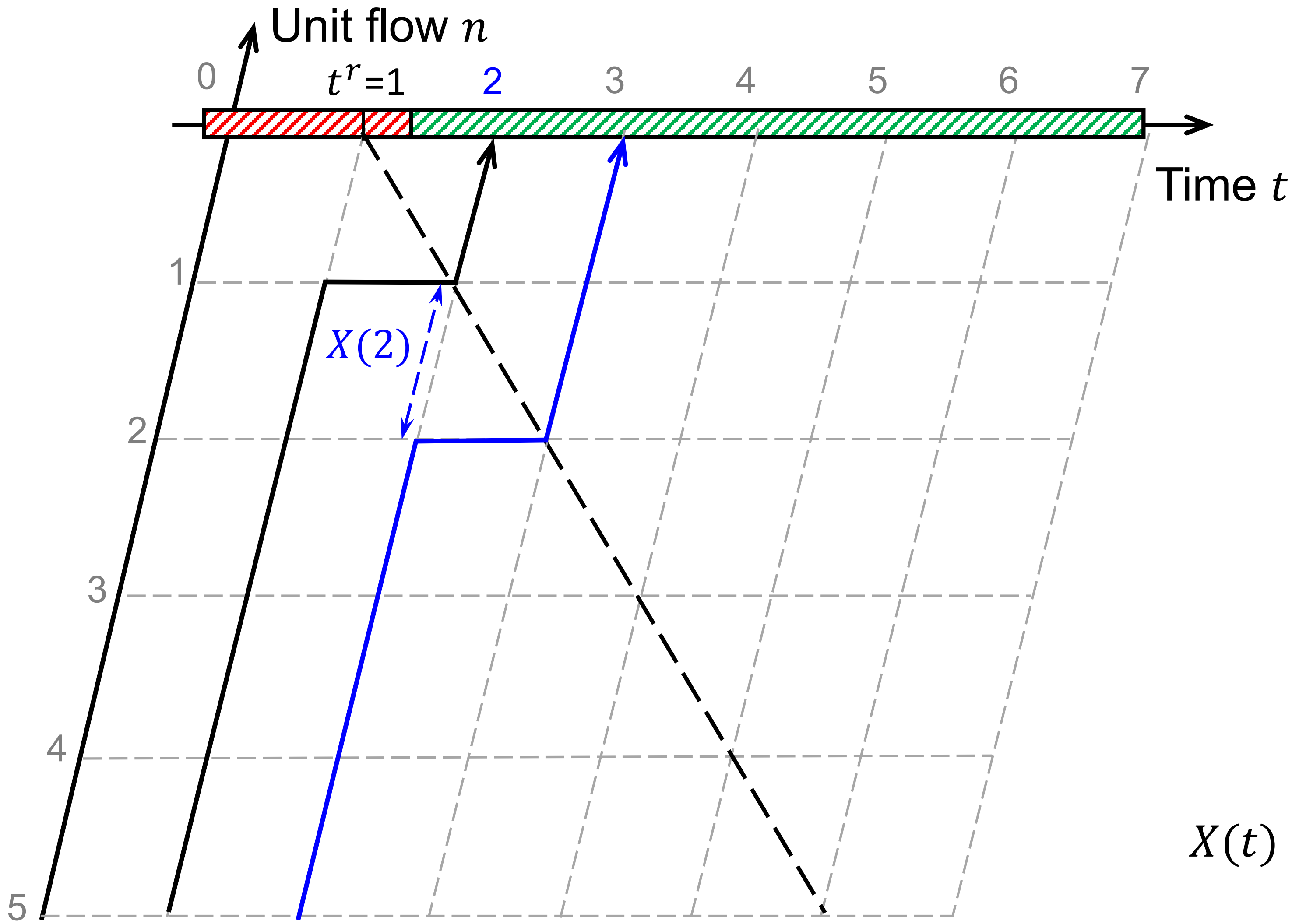
$$X(t) = \begin{cases} X^n(t) - \max\{0, t - t^r\}, & X^n(t) > 0 \\ 0, & X^n(t) = 0 \end{cases}$$

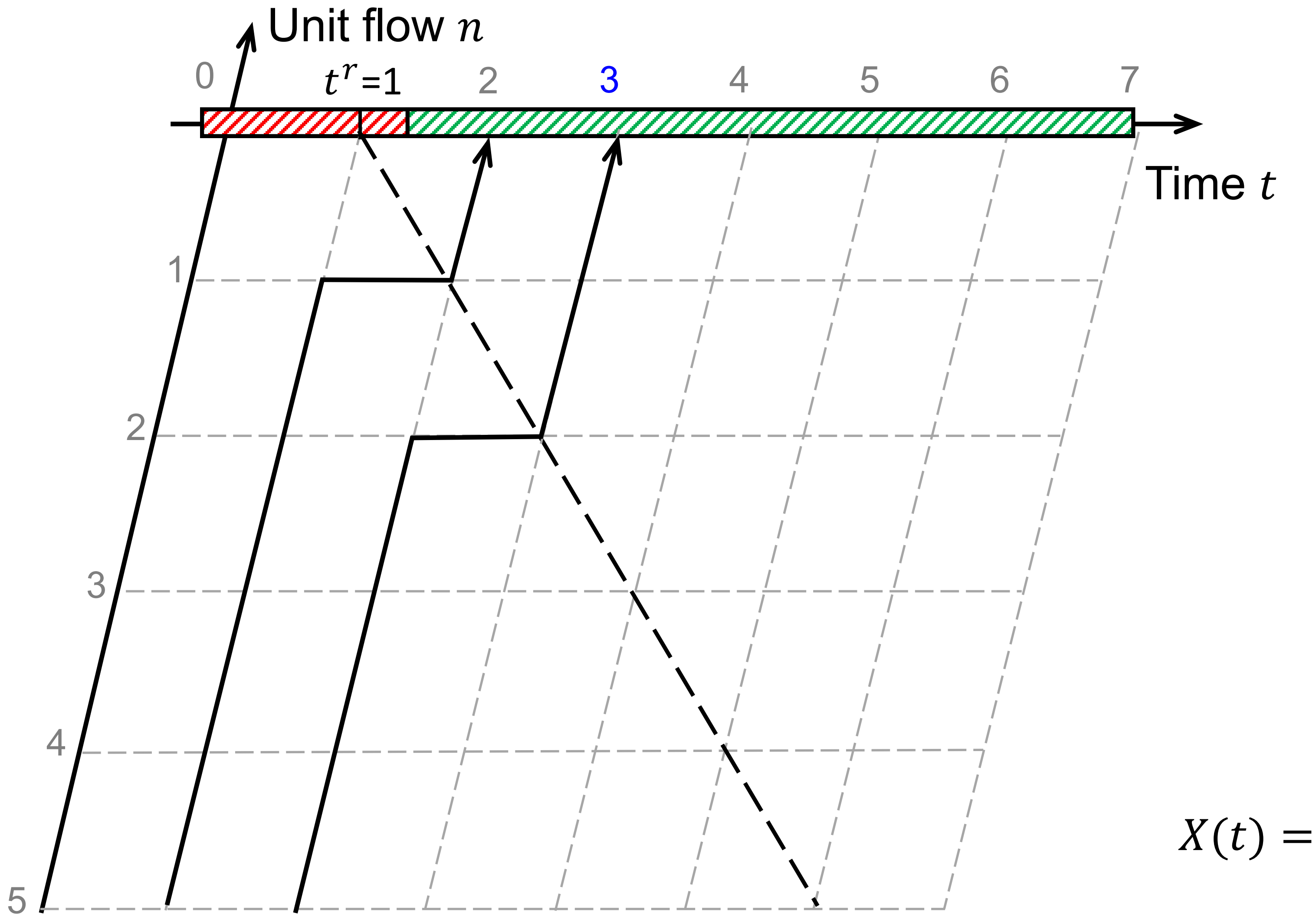
Point-Queue Under Newellian Coordinates



Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1	1	1	1	0
2	1	2	1	1
3				1
4				
5				
6				
7				

$$X(t) = \begin{cases} X^n(t) - \max\{0, t - t^r\}, & X^n(t) > 0 \\ 0, & X^n(t) = 0 \end{cases}$$

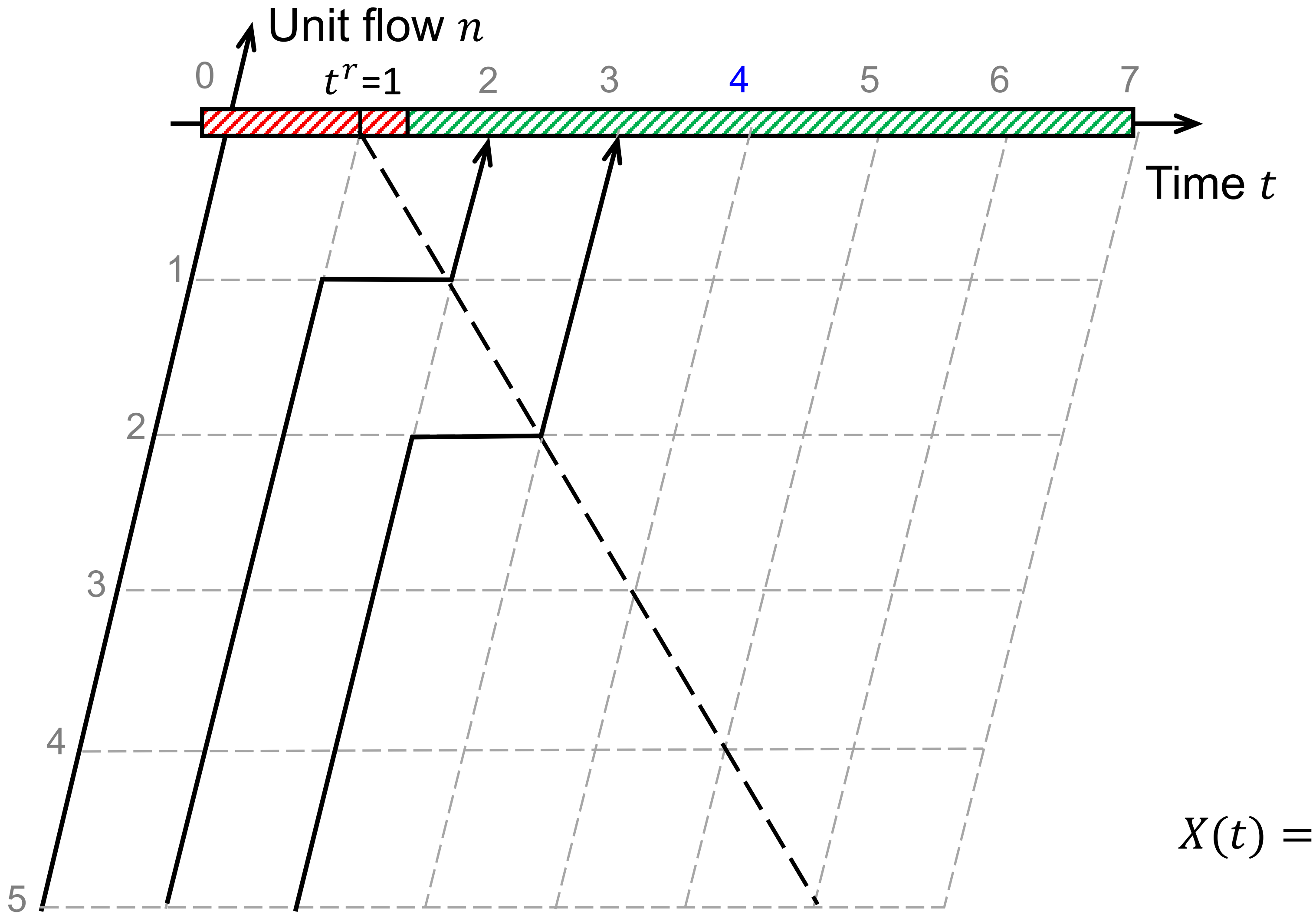
Point-Queue Under Newellian Coordinates



Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1	1	1	1	0
2	1	2	1	1
3	0	0	0	1
4				
5				
6				
7				

$$X(t) = \begin{cases} X^n(t) - \max\{0, t - t^r\}, & X^n(t) > 0 \\ 0, & X^n(t) = 0 \end{cases}$$

Point-Queue Under Newellian Coordinates



Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1	1	1	1	0
2	1	2	1	1
3	0	0	0	1
4	0	0	0	0
5				
6				
7				

$$X(t) = \begin{cases} X^n(t) - \max\{0, t - t^r\}, & X^n(t) > 0 \\ 0, & X^n(t) = 0 \end{cases}$$

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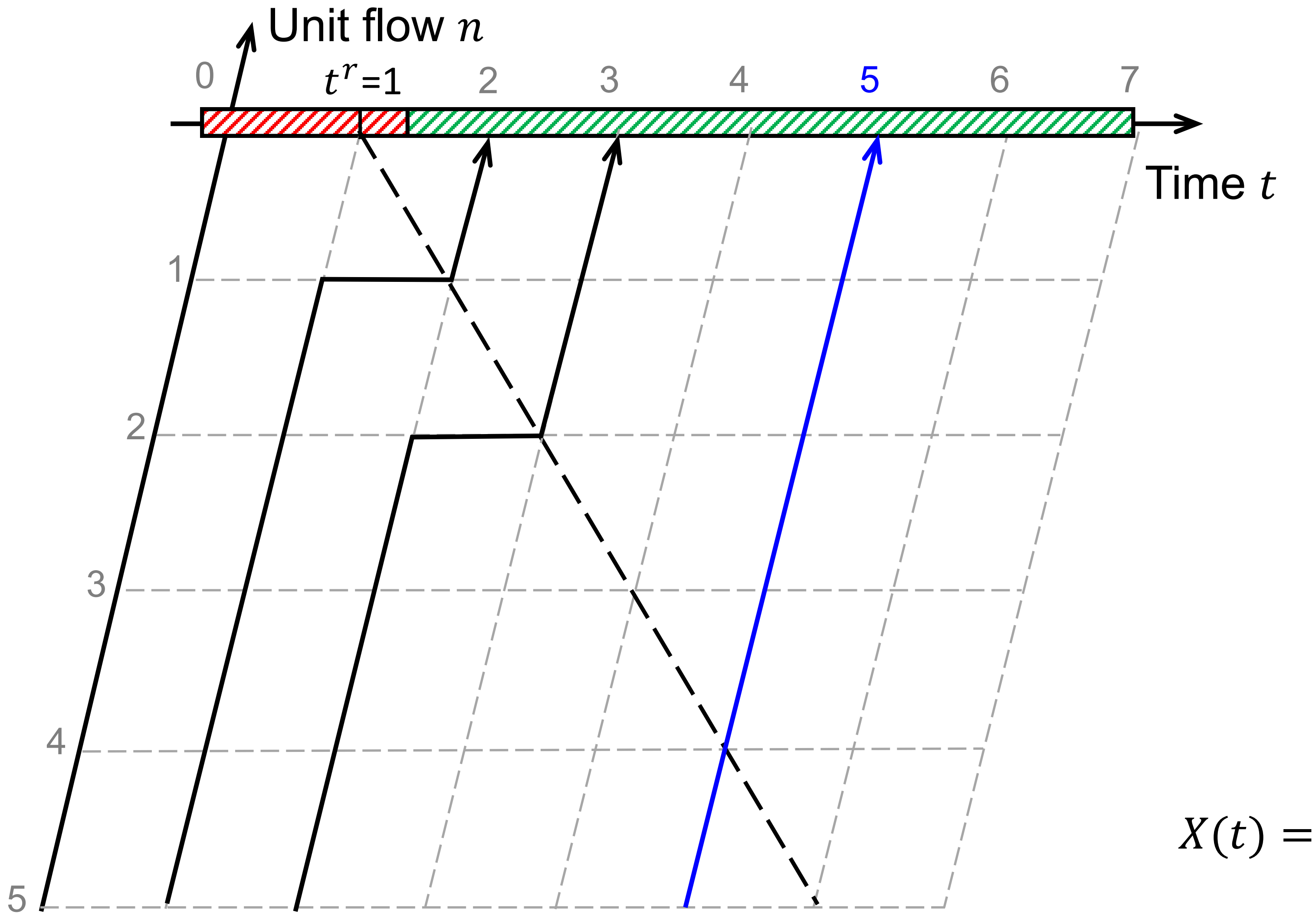
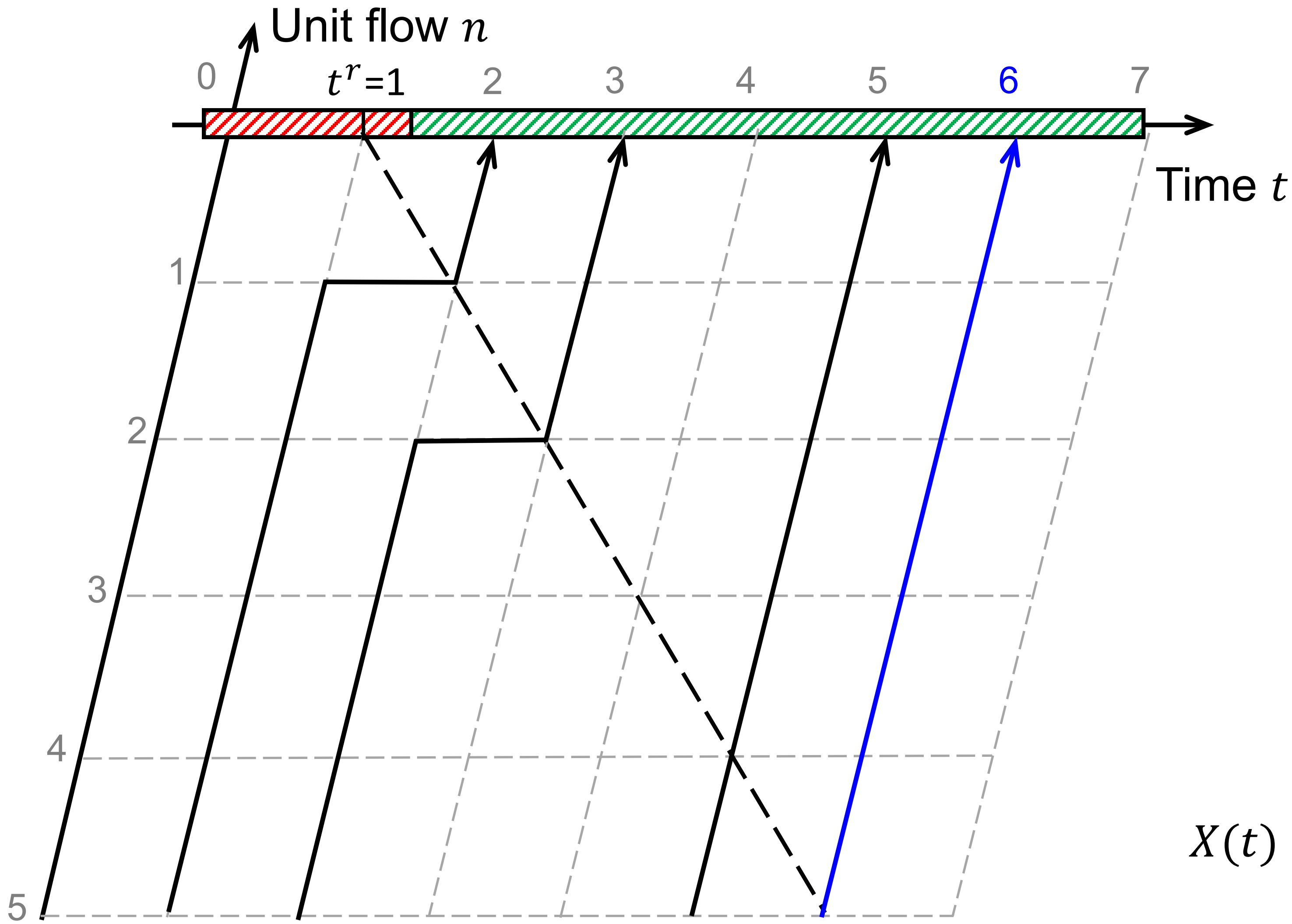


Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1	1	1	1	0
2	1	2	1	1
3	0	0	0	1
4	0	0	0	0
5	1	0	0	1
6				
7				

$$X(t) = \begin{cases} X^n(t) - \max\{0, t - t^r\}, & X^n(t) > 0 \\ 0, & X^n(t) = 0 \end{cases}$$

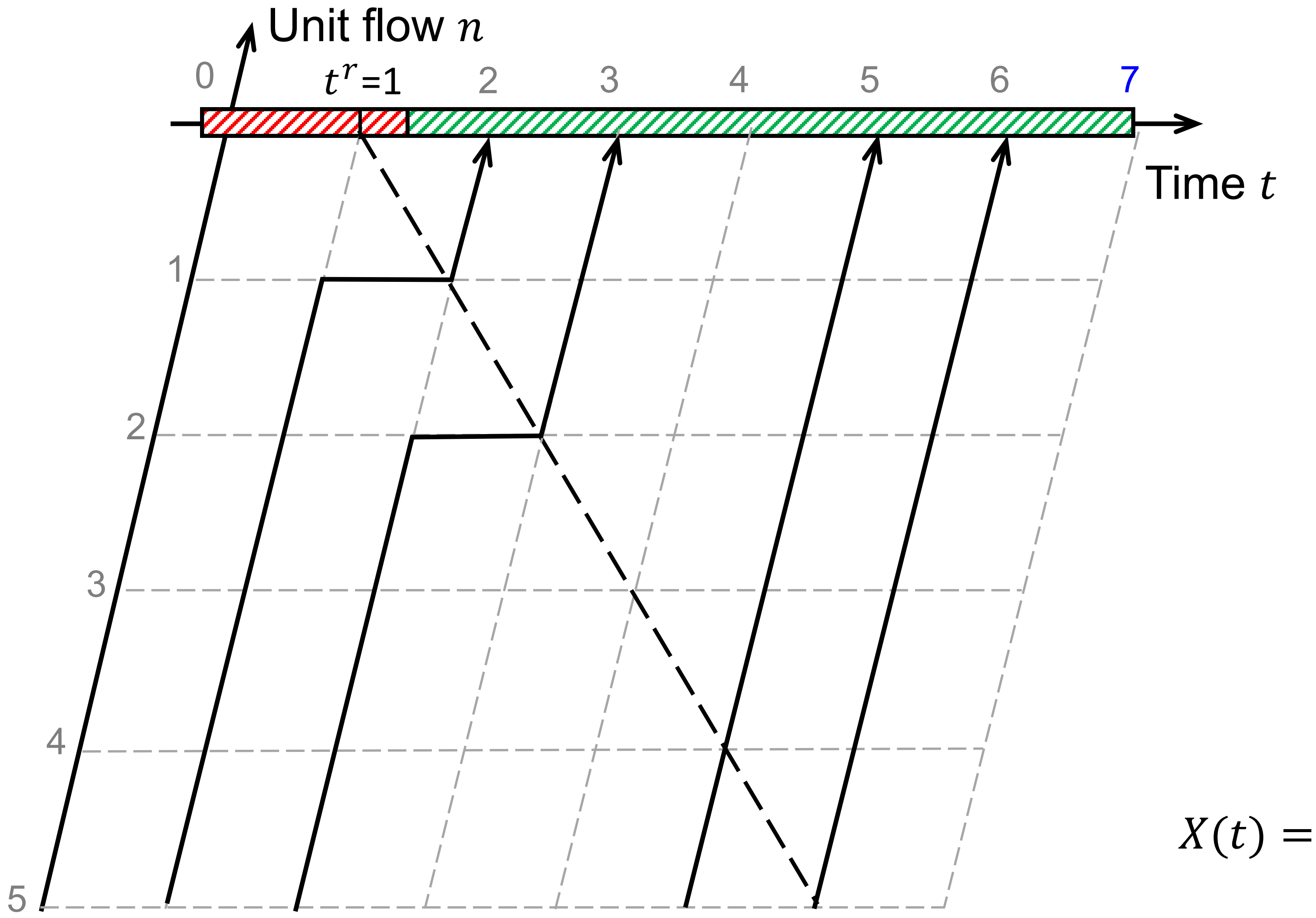
Point-Queue Under Newellian Coordinates



Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1	1	1	1	0
2	1	2	1	1
3	0	0	0	1
4	0	0	0	0
5	1	0	0	1
6	1	0	0	1
7				

$$X(t) = \begin{cases} X^n(t) - \max\{0, t - t^r\}, & X^n(t) > 0 \\ 0, & X^n(t) = 0 \end{cases}$$

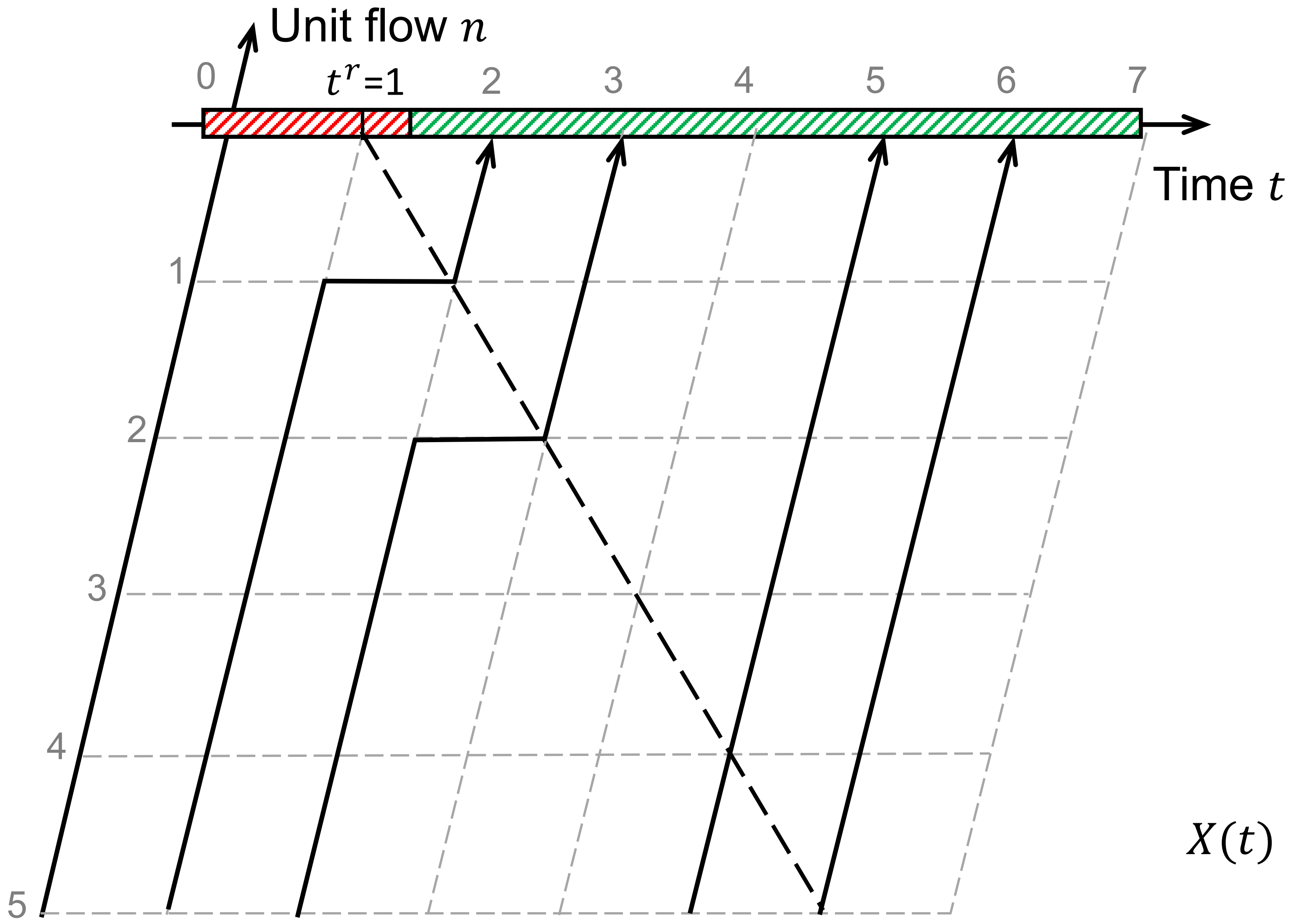
Point-Queue Under Newellian Coordinates



Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1	1	1	1	0
2	1	2	1	1
3	0	0	0	1
4	0	0	0	0
5	1	0	0	1
6	1	0	0	1
7	0	0	0	0

$$X(t) = \begin{cases} X^n(t) - \max\{0, t - t^r\}, & X^n(t) > 0 \\ 0, & X^n(t) = 0 \end{cases}$$

Point-Queue Under Newellian Coordinates



Time t	$A(t)$	$X^n(t)$	$X(t)$	$B(t)$
1	1	1	1	0
2	1	2	1	1
3	0	0	0	1
4	0	0	0	0
5	1	0	0	1
6	1	0	0	1
7	0	0	0	0

$$X(t) = \begin{cases} X^n(t) - \max\{0, t - t^r\}, & X^n(t) > 0 \\ 0, & X^n(t) = 0 \end{cases}$$

$$\boxed{X(t) = X(t-1) + A(t) - B(t)}$$

Outline

- ☐ Introduction of the Newellian coordinates
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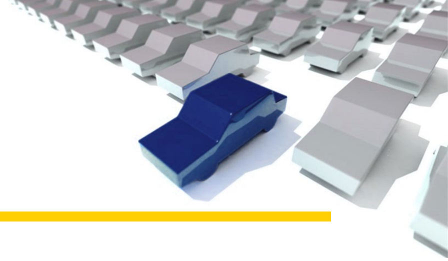
Stochastic point-queue model



- Point-queue model

$$X(t) = X(t-1) + A(t) - B(t)$$

- Stochastic point-queue model

$$X(t) = X(t-1) + A(t) - B(t) = X'(t) - B(t)$$

1) Stochastic arrival (Bernoulli distribution each time)

$$A(t) \sim \text{Bernoulli}\big(a(t)\big) \qquad \mathbb{P}(A(t) = 1) = a(t)$$

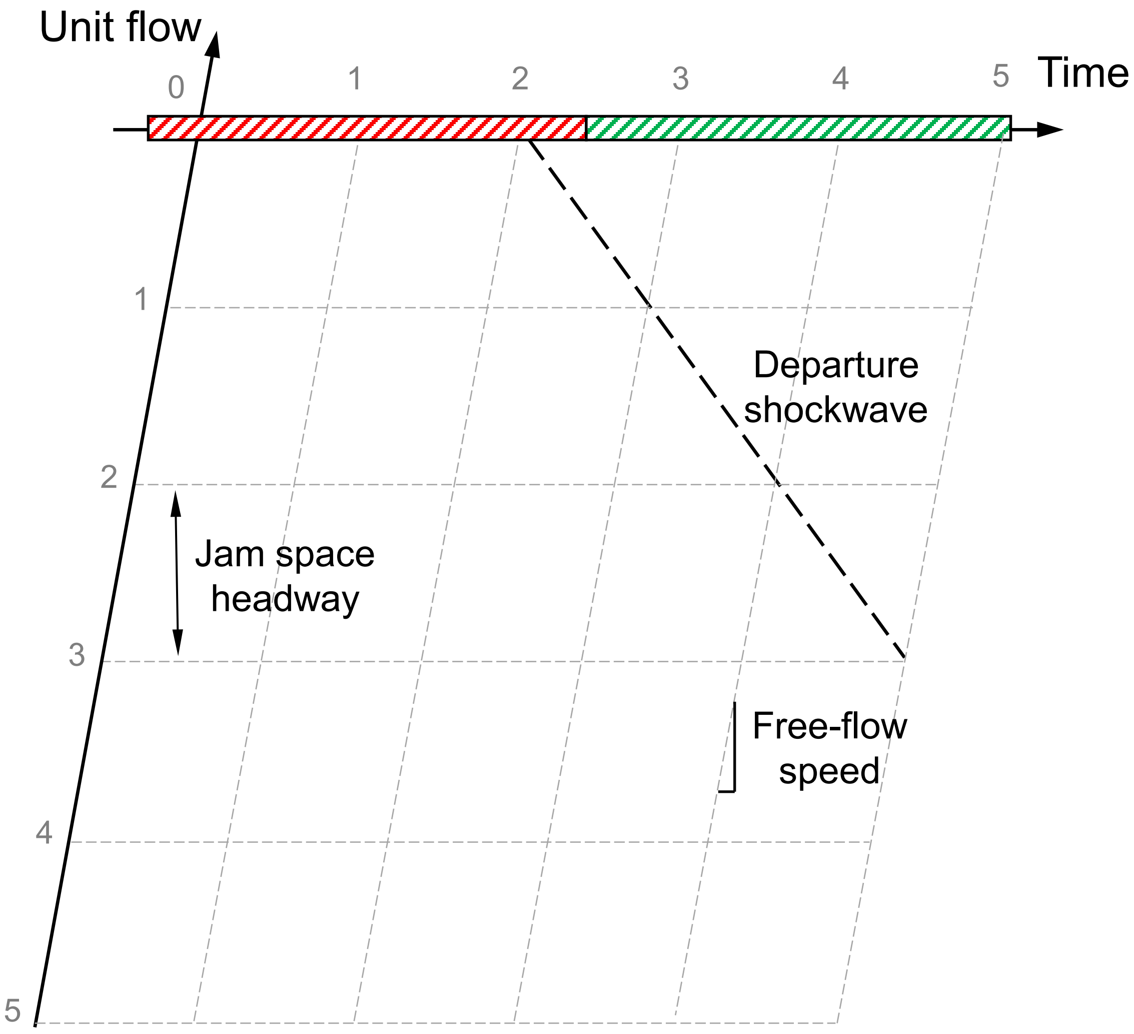
(A Poisson process when $\Delta t \to 0$)

2) Deterministic departure controlled by traffic signal

$$\mathbb{P}(B(t) = 1) = b(t) = \mathbb{P}(X(t) \geq 1 \,\&\, S(t) = 1)$$

Probabilistic Time-Space (PTS) Diagram

Time t	Arrival $a(t)$	Signal state $S(t)$	Queue length distribution $x(t,k)$				Departure $b(t)$
			0	1	2	3	
➡ 0			1.000	0.000	0.000	0.000	

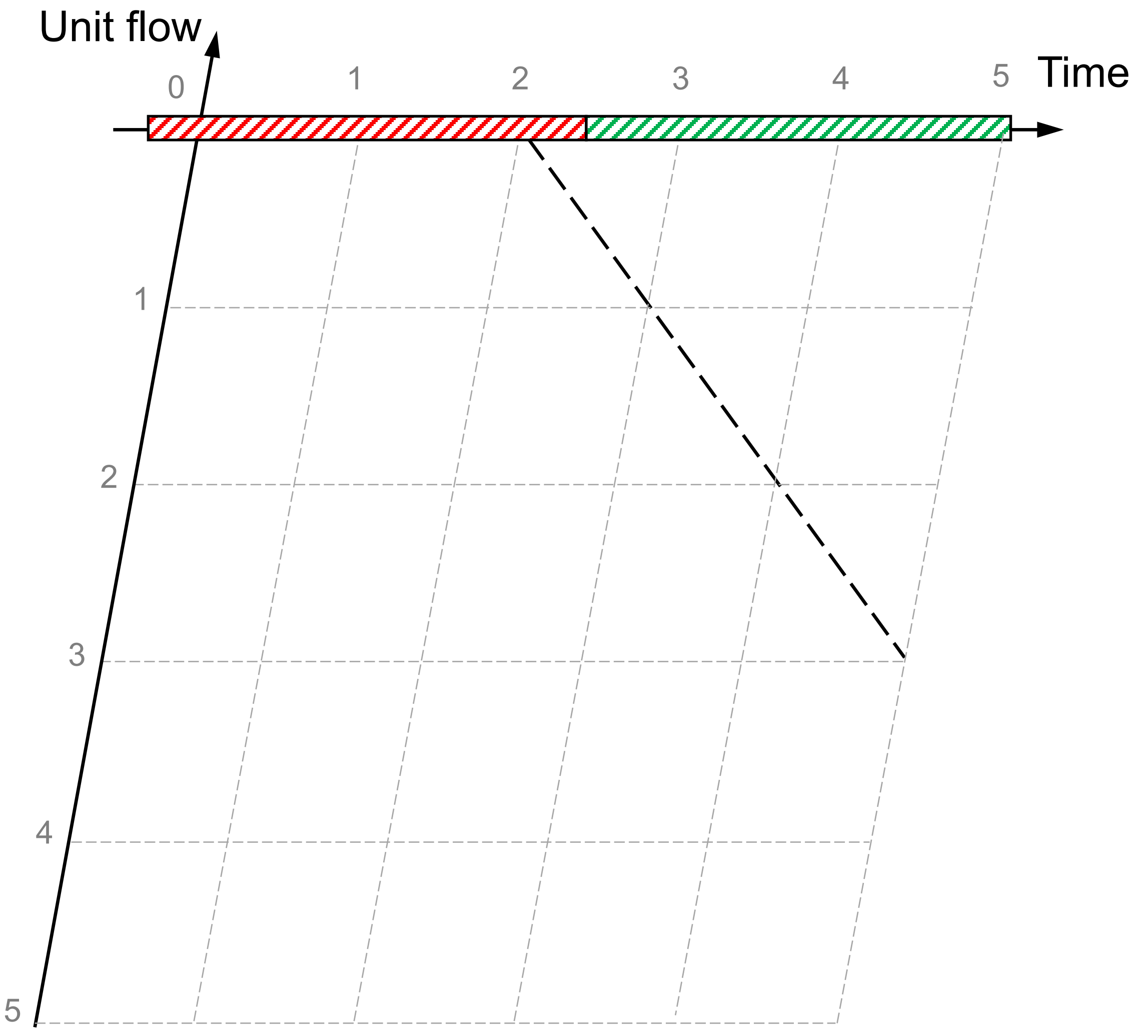


Probabilistic Time-Space (PTS) Diagram

Time t	Arrival $a(t)$	Signal state $S(t)$	Queue length distribution $x(t,k)$				Departure $b(t)$
			0	1	2	3	
0			1.000	0.000	0.000	0.000	
1	0.6		0.4 0.400	0.6 0.600	0.000	0.000	
		0					

Arrival step

$$x'(t,k+1) = x(t-1,k) \cdot a(t) + x(t-1,k+1) \cdot (1-a(t))$$



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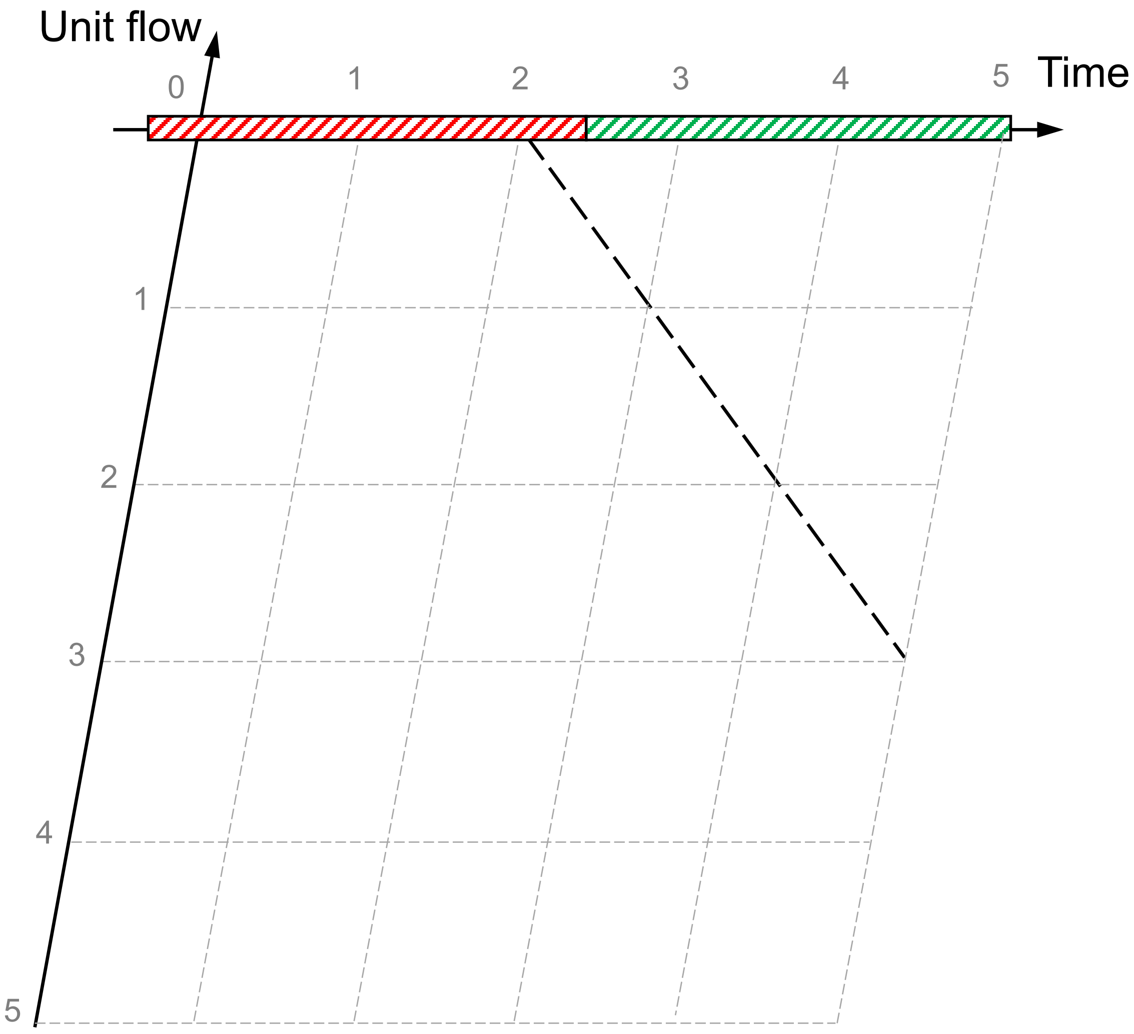
Time t	Arrival $a(t)$	Signal state $S(t)$	Queue length distribution $x(t,k)$				Departure $b(t)$
			0	1	2	3	
0			1.000	0.000	0.000	0.000	
1	0.6		0.400	0.600	0.000	0.000	
		0	0.400	0.600	0.000	0.000	0.000

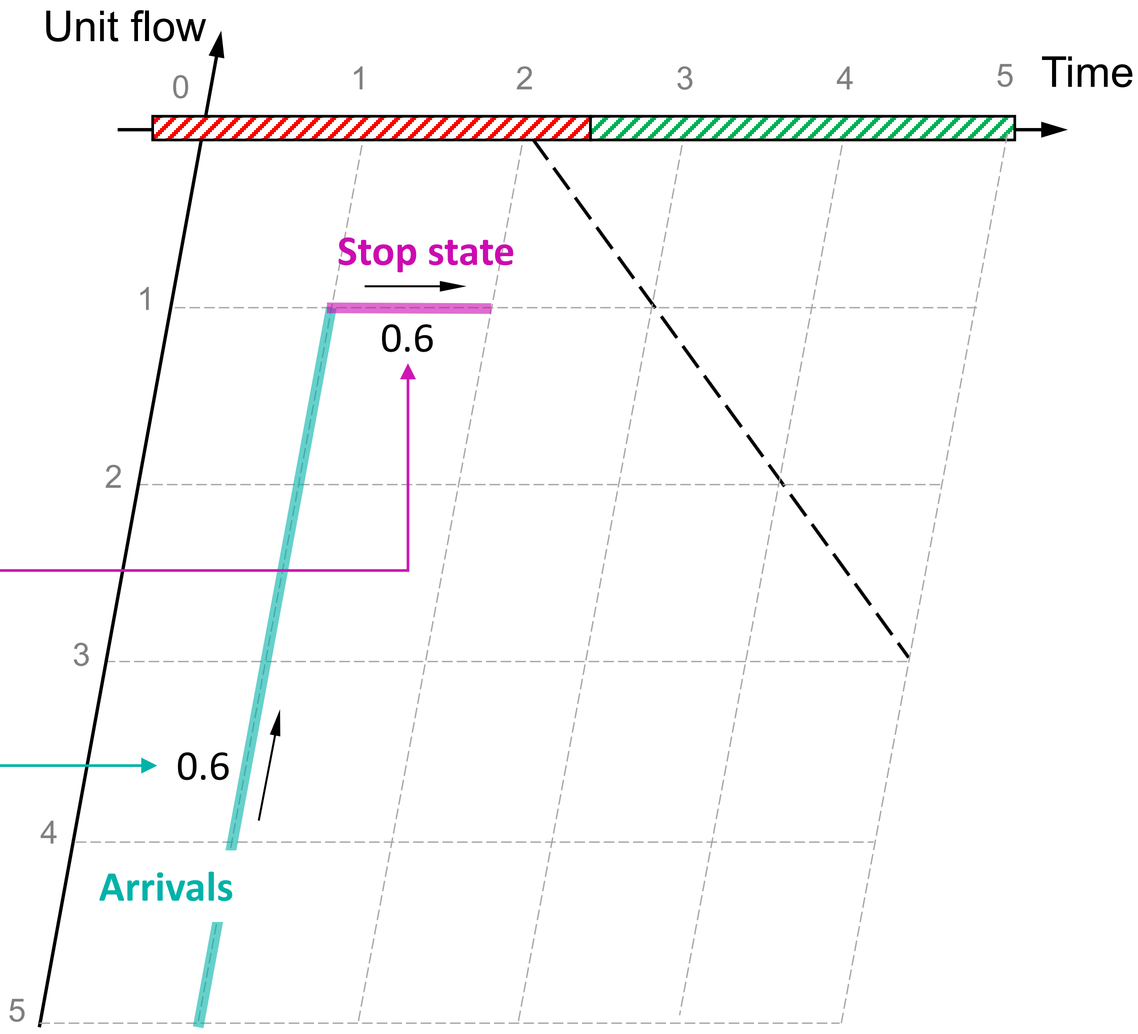
Departure step

$$\begin{cases} x(t,k) = x'(t,k+1)\cdot S(t) + x'(t,k)\cdot\big(1-S(t)\big), \forall k \geq 1 \\ x(t,0) = x'(t,1)\cdot S(t) + x'(t,0) \end{cases}$$



Probabilistic Time-Space (PTS) Diagram

Time t	**Arrival** $a(t)$	**Signal state** $S(t)$	**Queue length distribution** $x(t,k)$				**Departure** $b(t)$
			0	1	2	3	
0			1.000	0.000	0.000	0.000	
1	0.6		0.400	0.600	0.000	0.000	
		0	0.400	0.600	0.000	0.000	0.000

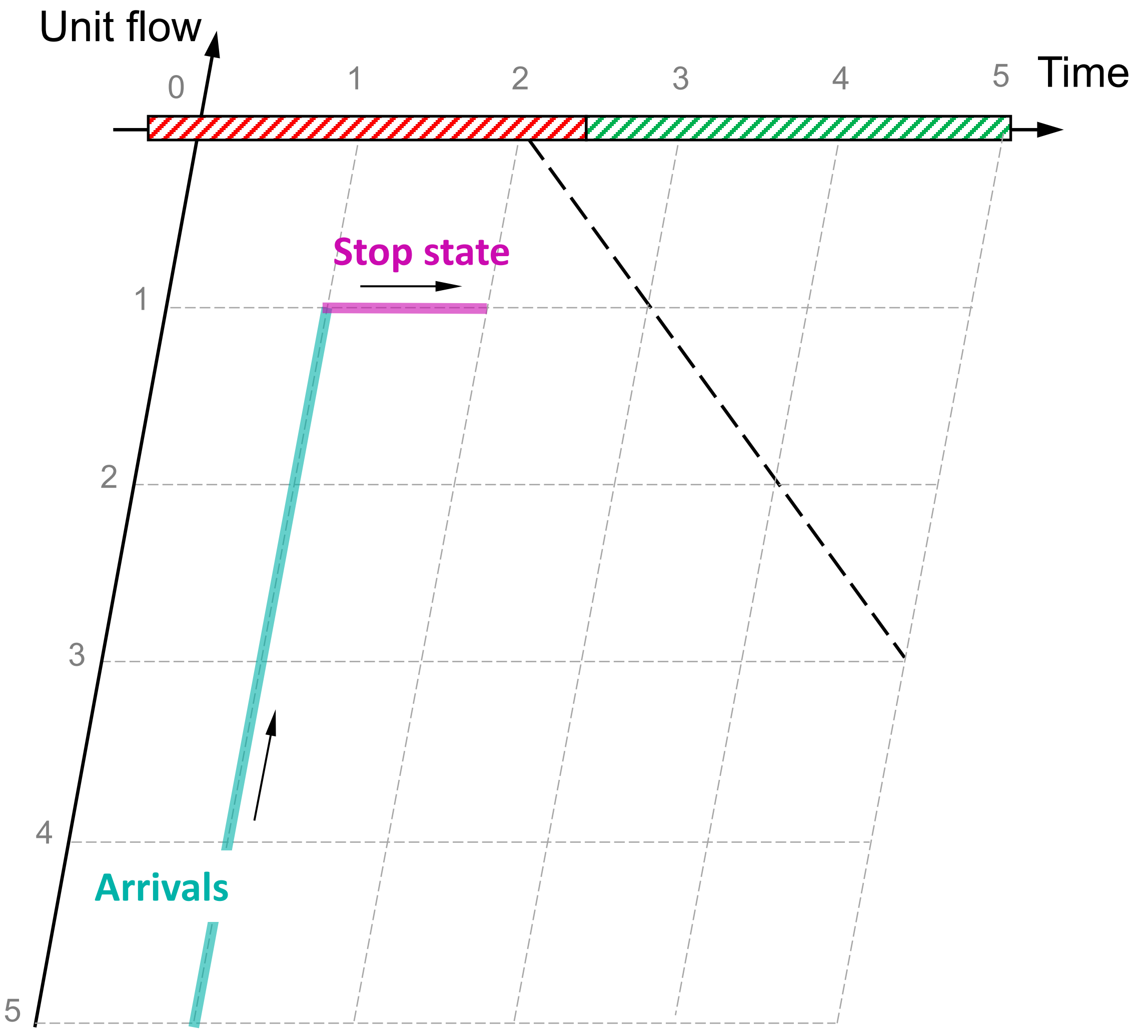


Probabilistic Time-Space (PTS) Diagram

Time t	**Arrival** $a(t)$	**Signal state** $S(t)$	**Queue length distribution** $x(t,k)$				**Departure** $b(t)$
			0	1	2	3	
0			1.000	0.000	0.000	0.000	
1	0.6		0.400	0.600	0.000	0.000	
		0	0.400	0.600	0.000	0.000	0.000
2	0.5		0.5 0.5 0.200	0.500	0.300	0.000	
		0					

Arrival step

$$x'(t,k+1) = x(t-1,k) \cdot a(t) + x(t-1,k+1) \cdot (1-a(t))$$

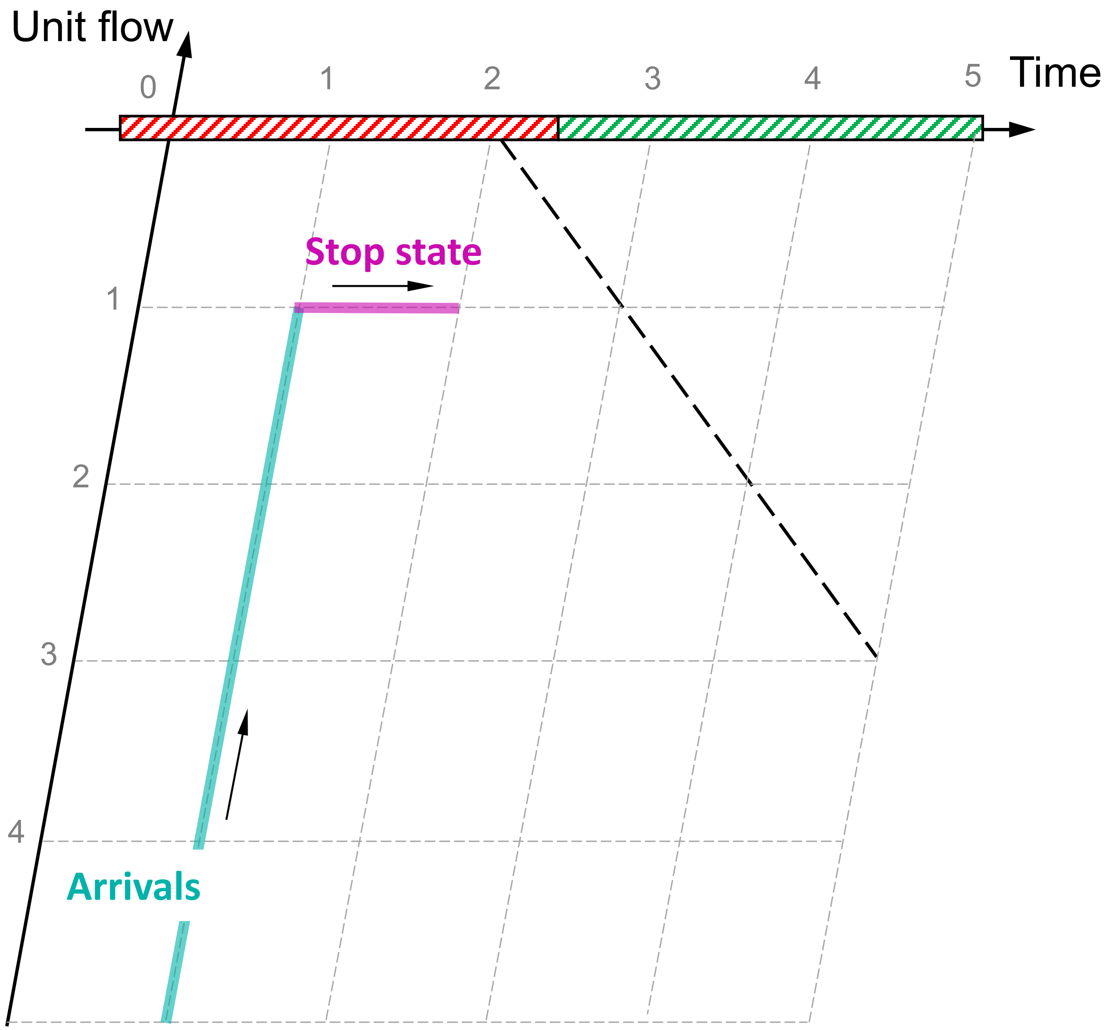


Probabilistic Time-Space (PTS) Diagram

Time t	Arrival $a(t)$	Signal state $S(t)$	Queue length distribution $x(t,k)$				Departure $b(t)$
			0	1	2	3	
0			1.000	0.000	0.000	0.000	
1	0.6		0.400	0.600	0.000	0.000	
		0	0.400	0.600	0.000	0.000	0.000
2	0.5		0.200	0.500	0.300	0.000	
			1 ↓ ↙ 0	↓ ↙	↓ ↙	↓	
		0	0.200	0.500	0.300	0.000	0.000

Departure step

$$\begin{cases} x(t,k) = x'(t,k+1)\cdot S(t) + x'(t,k)\cdot\big(1-S(t)\big), \forall k \geq 1 \\ x(t,0) = x'(t,1)\cdot S(t) + x'(t,0) \end{cases}$$

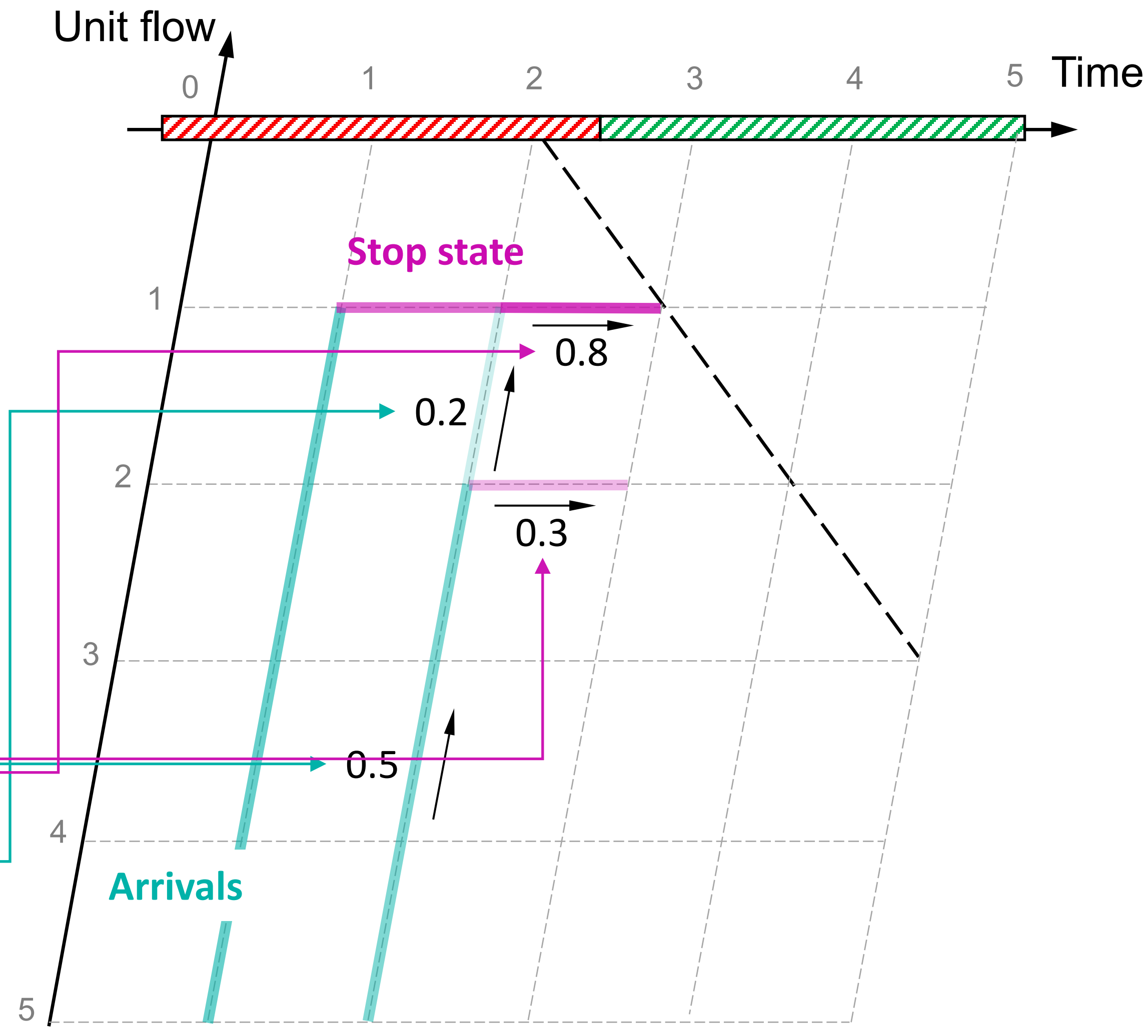


Probabilistic Time-Space (PTS) Diagram

Time t	Arrival $a(t)$	Signal state $S(t)$	Queue length distribution $x(t,k)$				Departure $b(t)$
			0	1	2	3	
0			1.000	0.000	0.000	0.000	
1	0.6		0.400	0.600	0.000	0.000	
		0	0.400	0.600	0.000	0.000	0.000
2	0.5		0.200	0.500	0.300	0.000	
		0	0.200	0.500	0.300	0.000	0.000

0.5

0.2



Probabilistic Time-Space (PTS) Diagram

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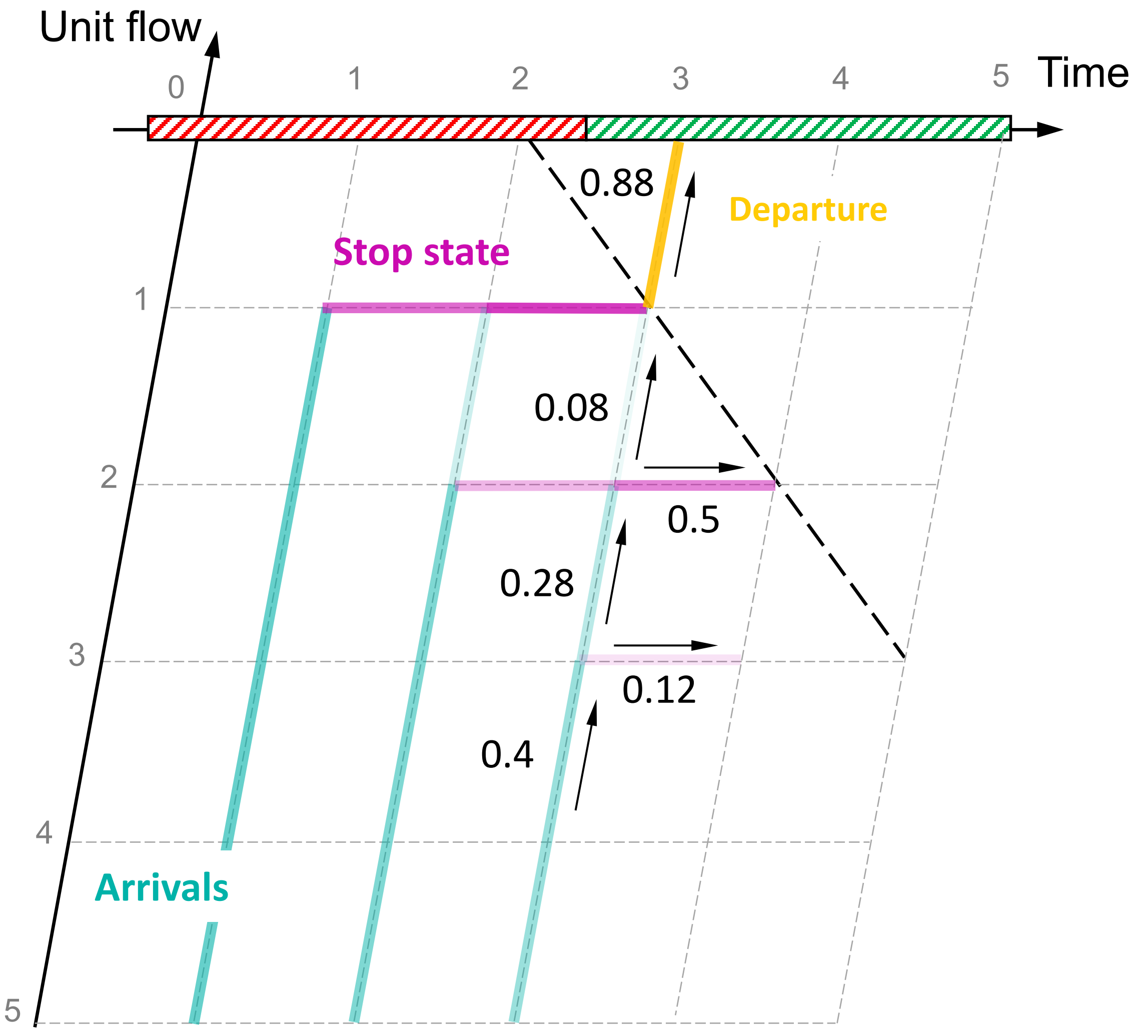
			0	1	2	3	
0			1.000	0.000	0.000	0.000	
1	0.6		0.400	0.600	0.000	0.000	
		0	0.400	0.600	0.000	0.000	0.000
2	0.5		0.200	0.500	0.300	0.000	
		0	0.200	0.500	0.300	0.000	0.000
3	0.4		0.120	0.380	0.380	0.120	
		1	0.500	0.380	0.120	0.000	0.880

0 1

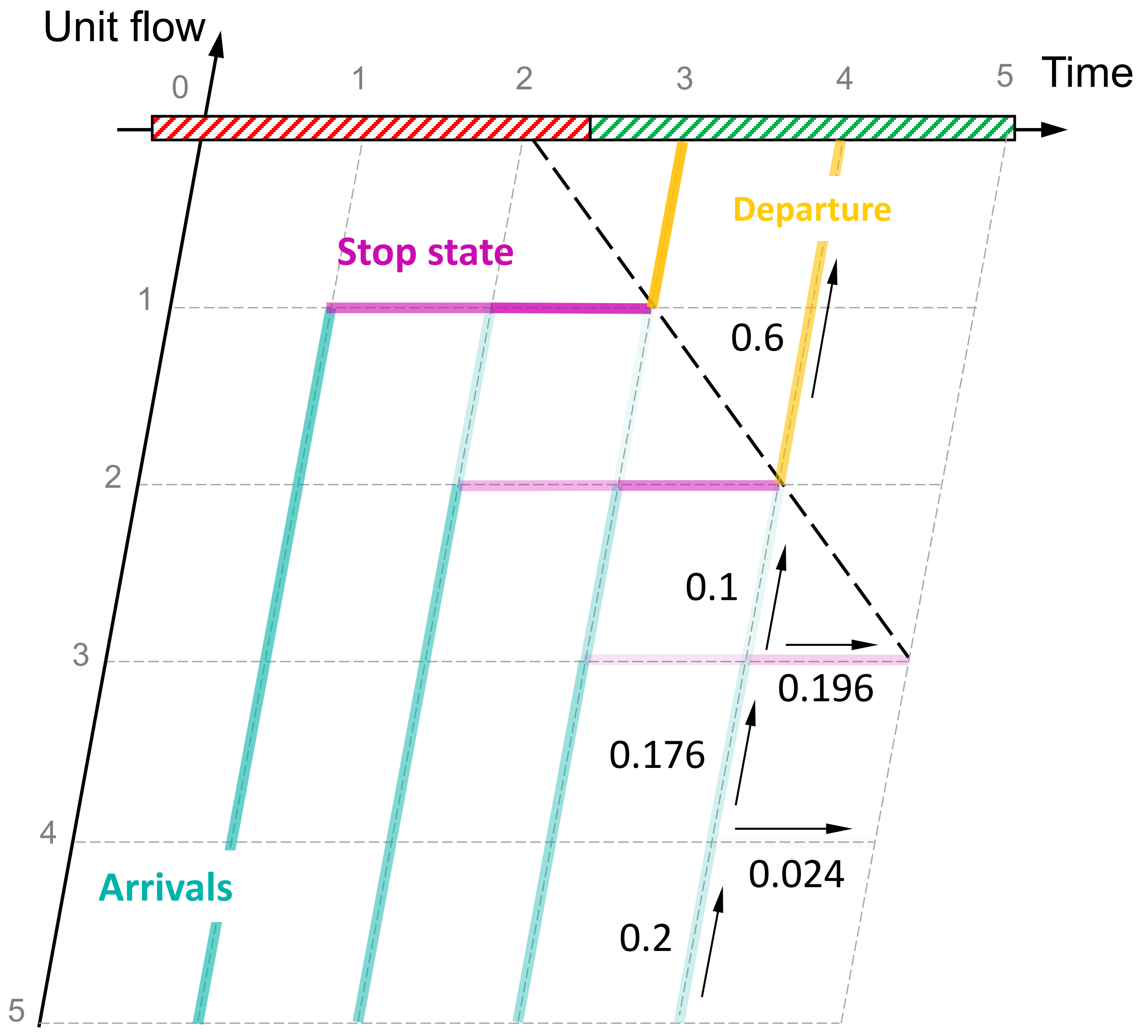
0.380 + 0.380 + 0.120

$$b(t) = \sum_{k=1}^{\infty} x'(t,k) \cdot S(t)$$



Probabilistic Time-Space (PTS) Diagram

Time t	**Arrival** $a(t)$	**Signal state** $S(t)$	**Queue length distribution** $x(t,k)$				**Departure** $b(t)$
			0	1	2	3	
0			1.000	0.000	0.000	0.000	
1	0.6		0.400	0.600	0.000	0.000	
		0	0.400	0.600	0.000	0.000	0.000
2	0.5		0.200	0.500	0.300	0.000	
		0	0.200	0.500	0.300	0.000	0.000
3	0.4		0.120	0.380	0.380	0.120	
		1	0.500	0.380	0.120	0.000	0.880
4	0.2		0.400	0.404	0.172	0.024	
		1	0.804	0.172	0.024	0.000	0.600



Probabilistic Time-Space (PTS) Diagram

Time t	Arrival $a(t)$	Signal state $S(t)$	Queue length distribution $x(t,k)$				Departure $b(t)$
			0	1	2	3	
0			1.000	0.000	0.000	0.000	
1	0.6		0.400	0.600	0.000	0.000	
		0	0.400	0.600	0.000	0.000	0.000
2	0.5		0.200	0.500	0.300	0.000	
		0	0.200	0.500	0.300	0.000	0.000
3	0.4		0.120	0.380	0.380	0.120	
		1	0.500	0.380	0.120	0.000	0.880
4	0.2		0.400	0.404	0.172	0.024	
		1	0.804	0.172	0.024	0.000	0.600

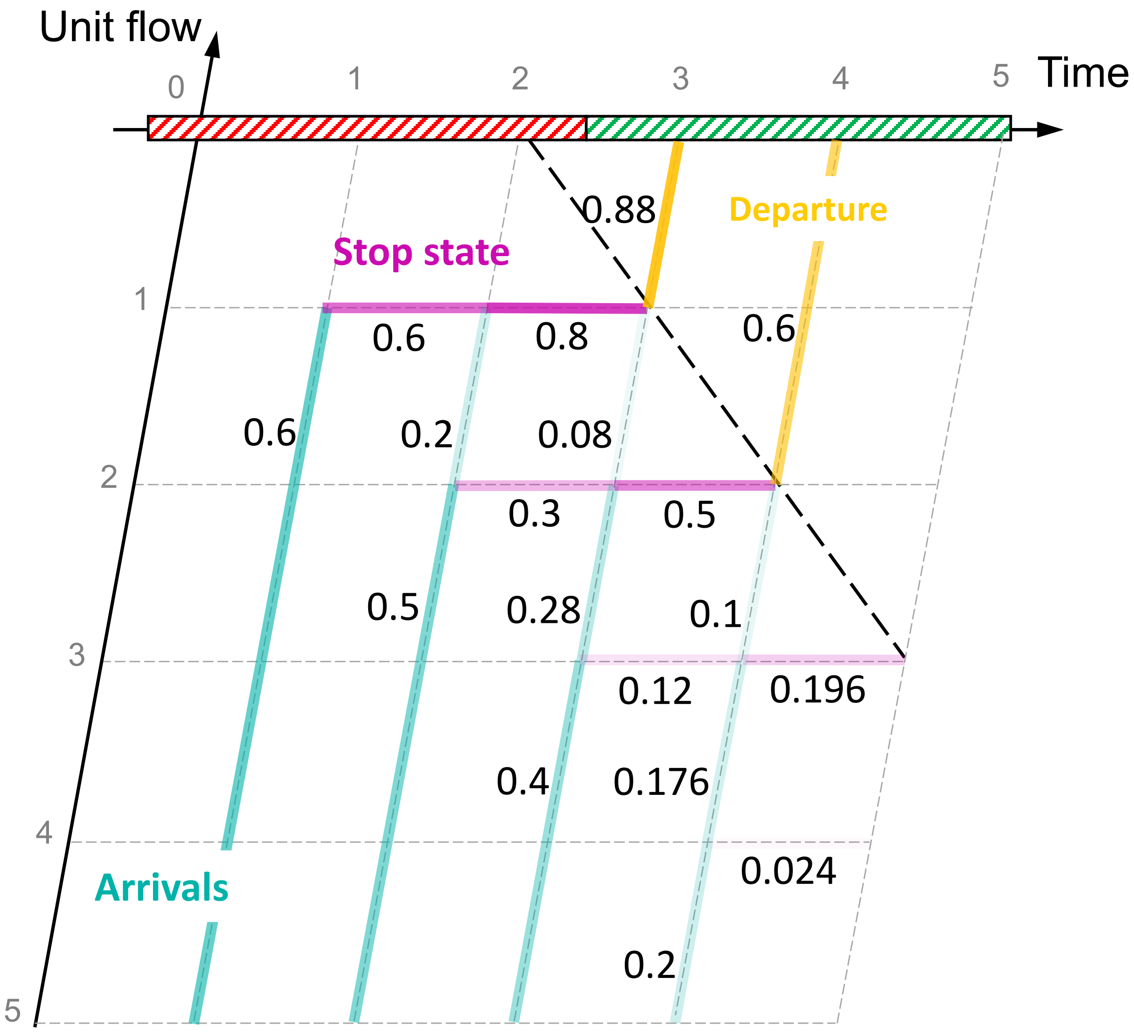
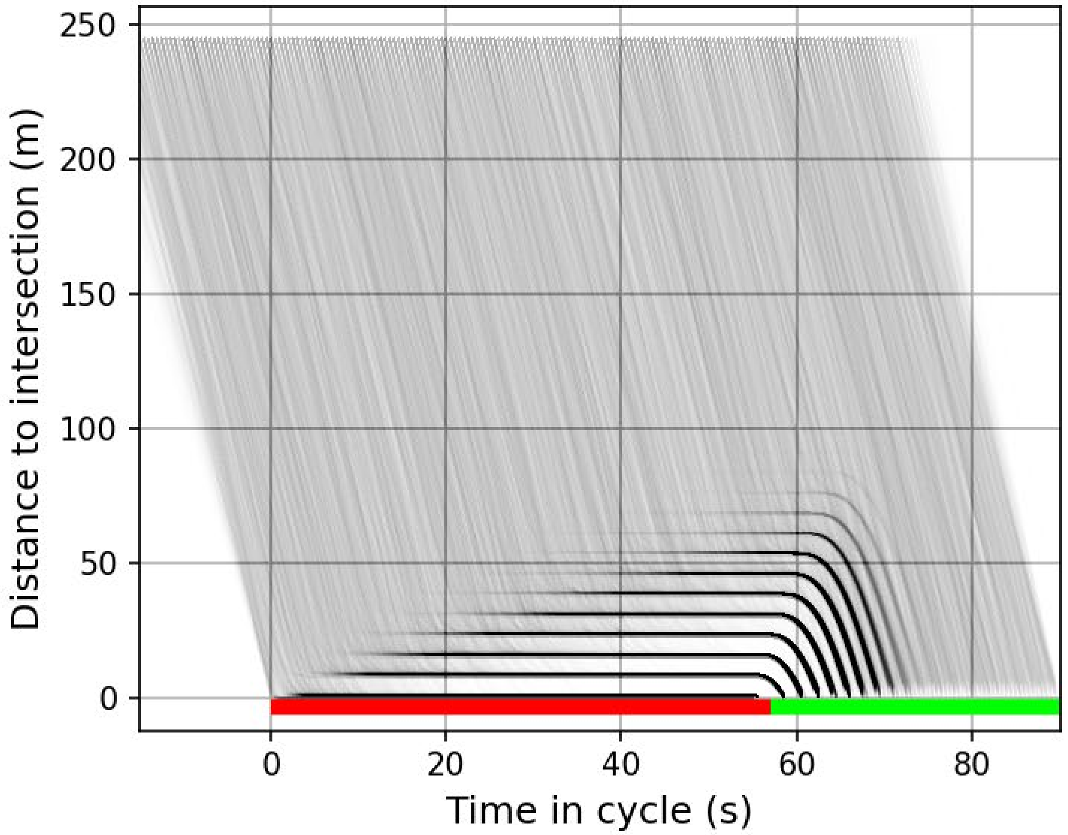


Illustration of the PTS diagram

Aggregated Time-space diagram
(from multiple cycles)



Probabilistic time-space (PTS) diagram

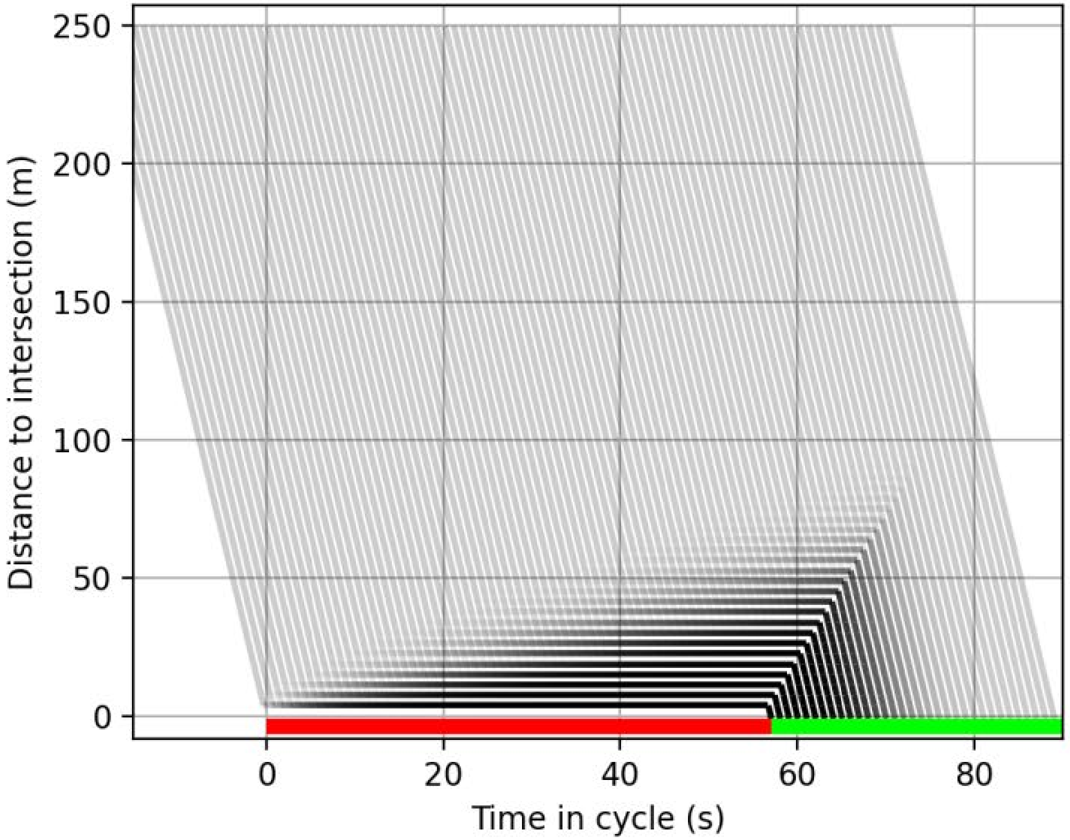
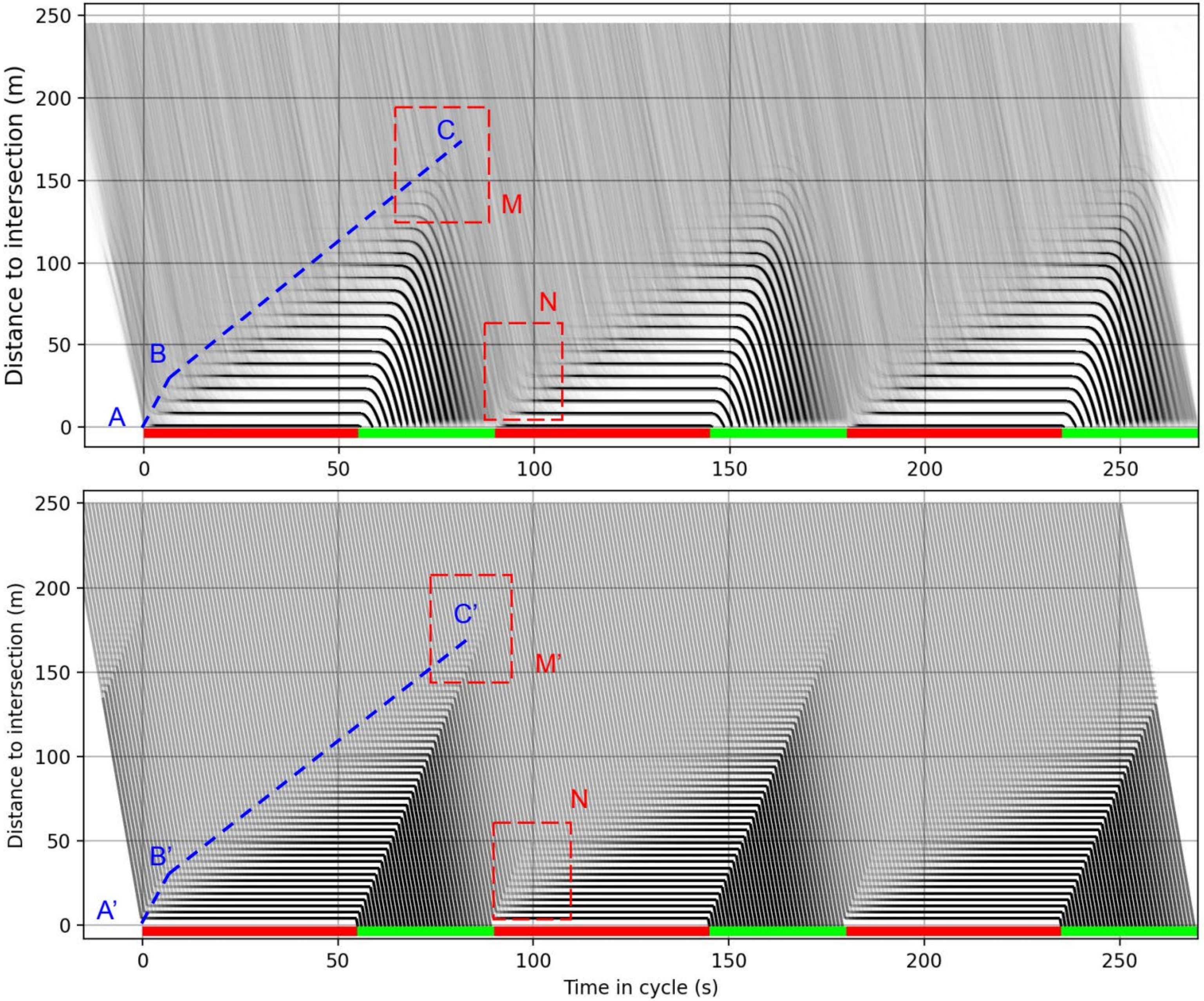


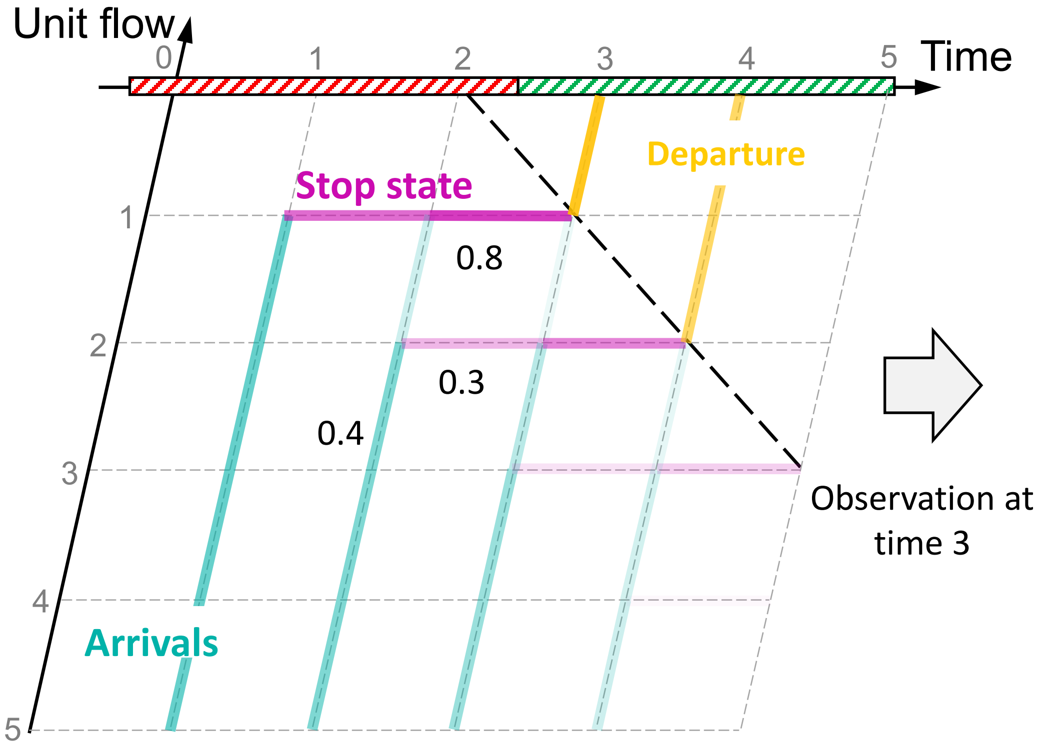
Illustration of the PTS diagram

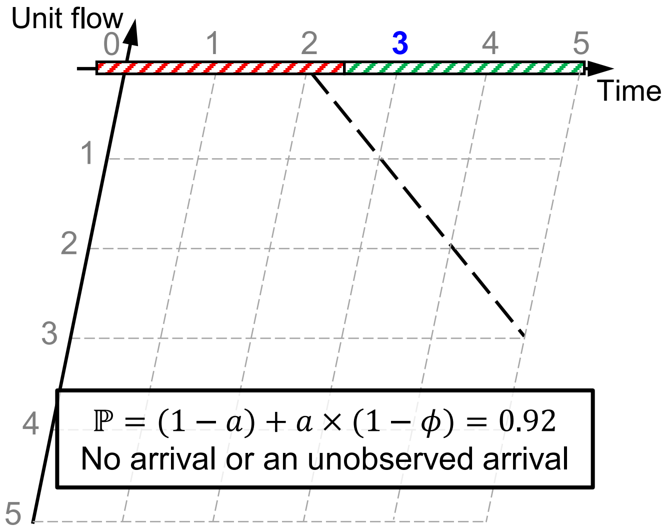


Outline

- ☐ Introduction of the Newellian coordinates
- ☐ Point-queue encoding of vehicle trajectories
- ☐ Probabilistic time-space diagram
- ☐ Parameter calibration and traffic signal retiming in the field

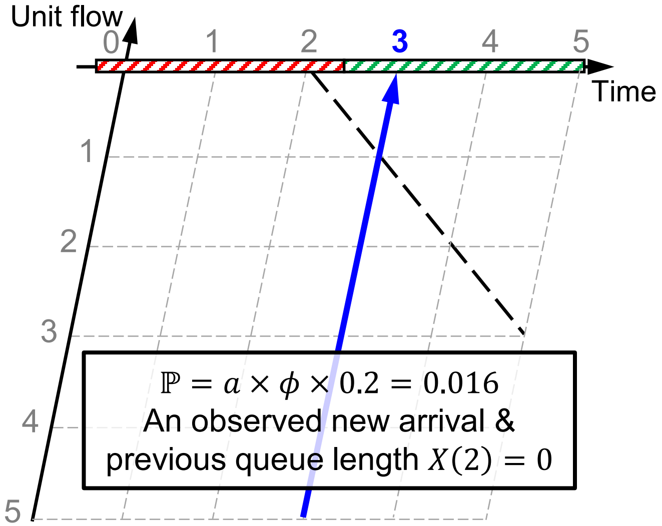
PTS Diagram & Observed Vehicle Trajectories





$\mathbb{P} = (1 - a) + a \times (1 - \phi) = 0.92$
No arrival or an unobserved arrival

Case 1: No observed trajectory



$\mathbb{P} = a \times \phi \times 0.2 = 0.016$
An observed new arrival &
previous queue length $X(2) = 0$

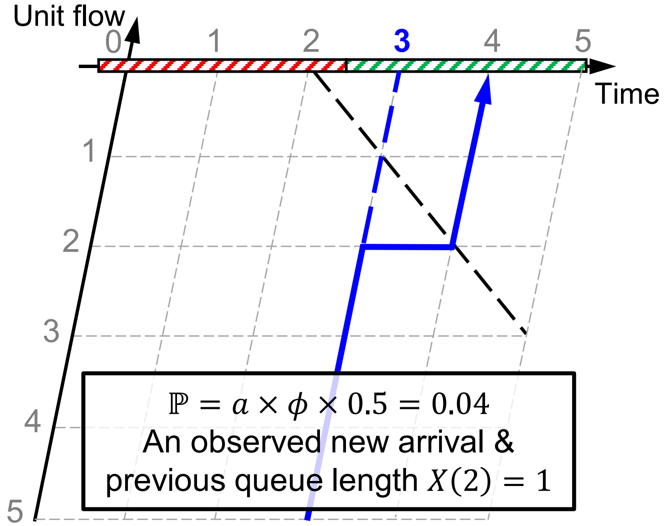
Case 2: Observed trajectory directly passed the intersection

By taking $t = 3$ as an example:

- Arrival probability $a = 0.4$
- Queue length distribution at $t = 2$:

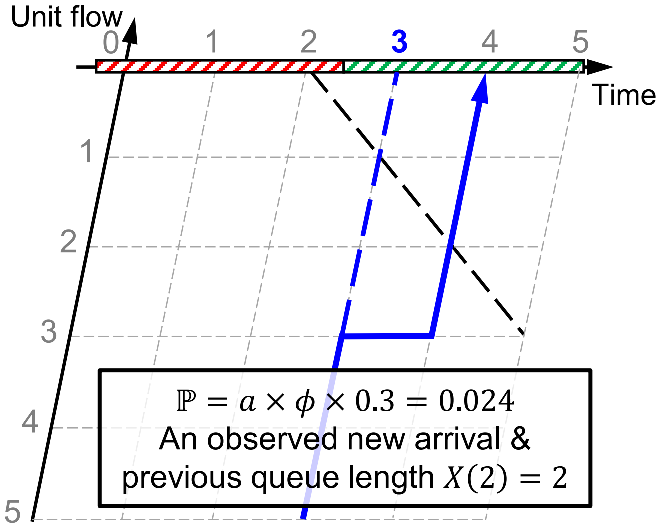
$X(2)$	0	1	2	3
Probability	0.2	0.5	0.3	0

- Penetration rate $\phi = 20\%$



$\mathbb{P} = a \times \phi \times 0.5 = 0.04$
An observed new arrival &
previous queue length $X(2) = 1$

Case 3: Observed trajectory stopped at location 2



$\mathbb{P} = a \times \phi \times 0.3 = 0.024$
An observed new arrival &
previous queue length $X(2) = 2$

Case 4: Observed trajectory stopped at location 3

PTS Diagram & Observed Vehicle Trajectories

- ❑ What is the probability $\mathbb{P}$ here?

$$\mathbb{P}(\mathcal{O}(t)|\Theta, X(t-1))$$

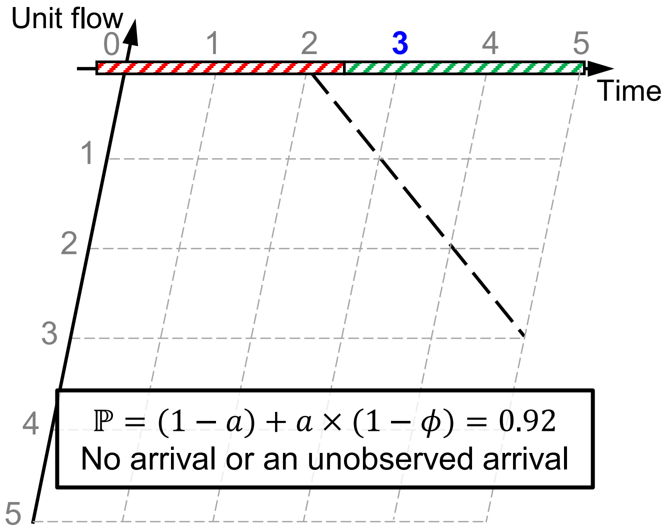
$\mathcal{O}(t)$	Observation
$\Theta = (\alpha, \phi)$	Traffic parameters
$X(t)$	Real-time traffic state

likelihood given traffic state and parameters

- ❑ Maximum likelihood estimation

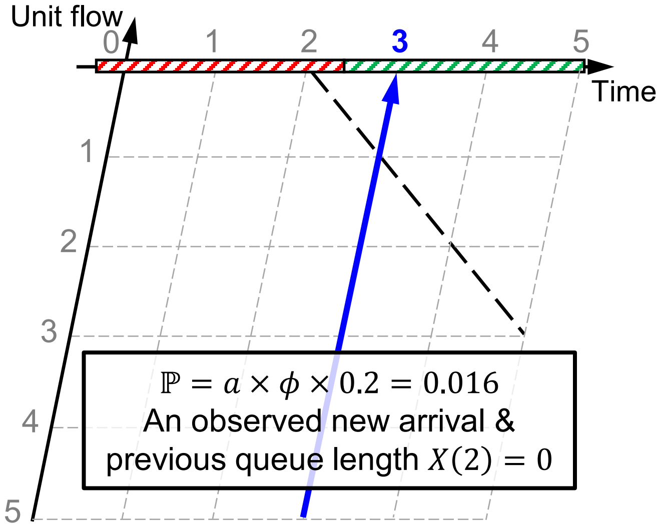
$$\widehat{\Theta} = \arg\max_{\Theta} \mathbb{P}(\mathcal{O}(1{:}T)|\Theta)$$

(find the parameter to maximize the probability that you have the given observed trajectories)



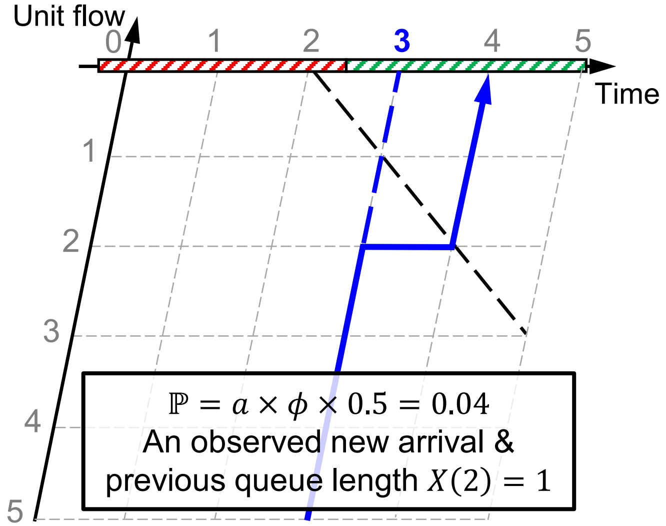
$\mathbb{P} = (1-a) + a \times (1-\phi) = 0.92$
No arrival or an unobserved arrival

Case 1: No observed trajectory



$\mathbb{P} = a \times \phi \times 0.2 = 0.016$
An observed new arrival &
previous queue length $X(2) = 0$

Case 2: Observed trajectory directly passed the intersection



$\mathbb{P} = a \times \phi \times 0.5 = 0.04$
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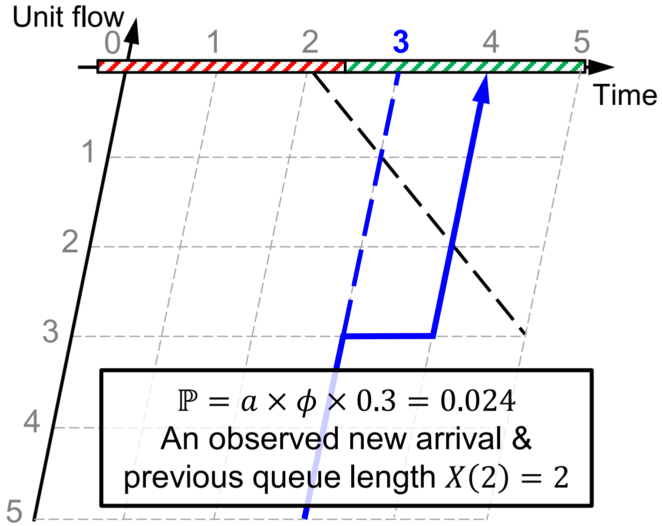
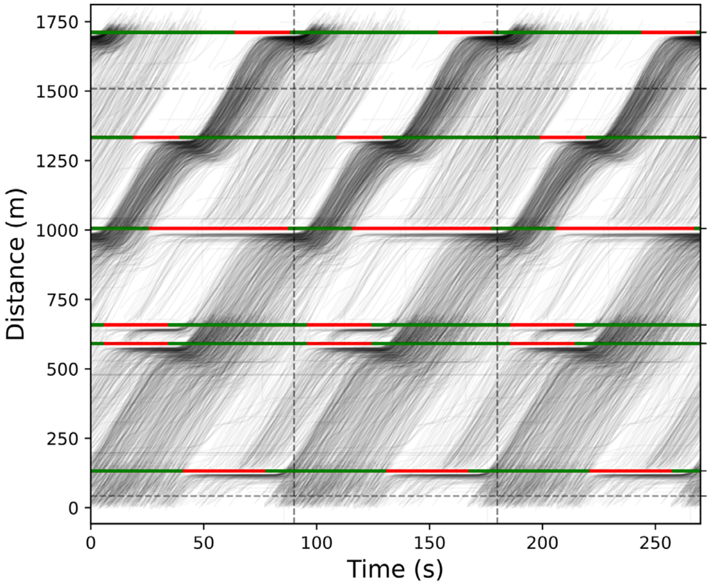


$\mathbb{P} = a \times \phi \times 0.3 = 0.024$
An observed new arrival &
previous queue length $X(2) = 2$

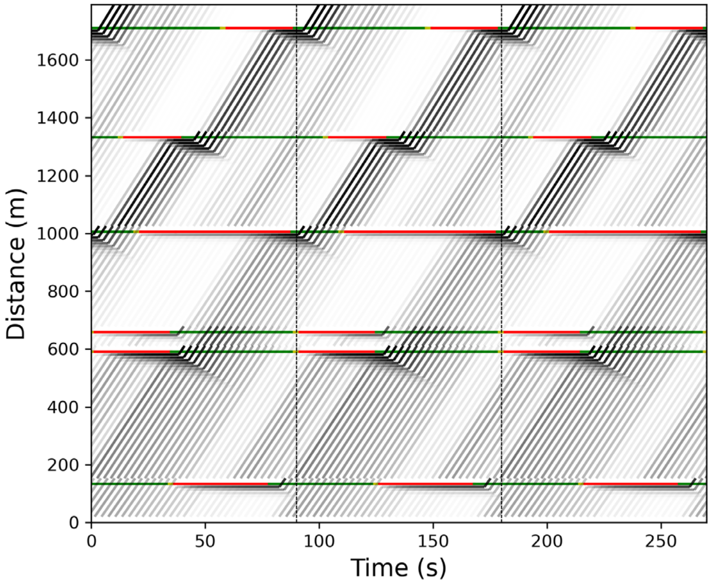
Case 4: Observed trajectory stopped at location 3

Traffic State Reconstruction for a Corridor

Aggregated time-space diagram of the corridor



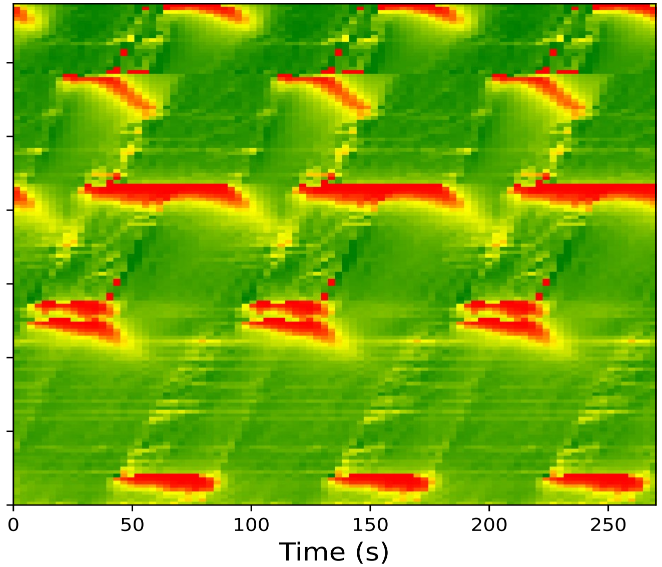
Traffic state & Parameter estimation



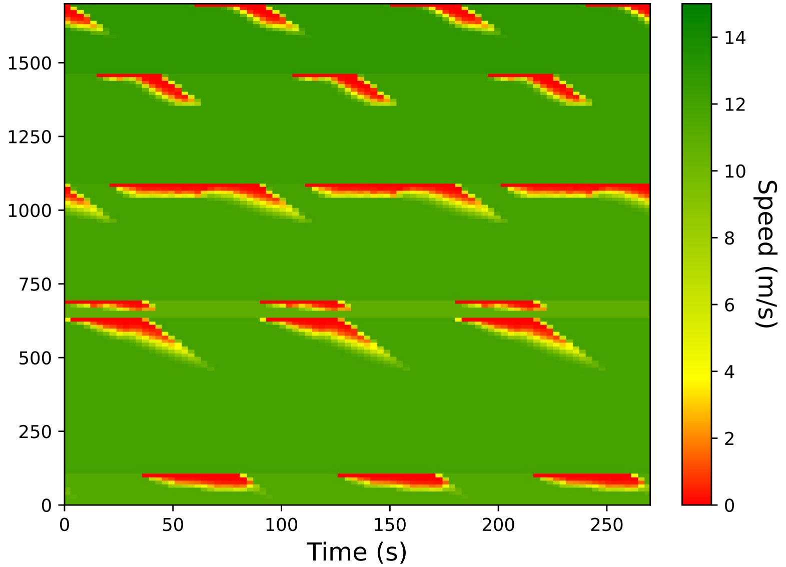
“Probabilistic time-space diagram” of the corridor generated from the queueing model

Space-mean speed calculation

Space-mean speed heatmap

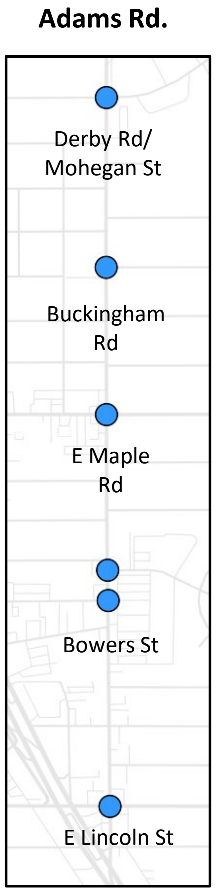


Similar pattern & queue profile



Model-estimated space-mean speed heatmap

Traffic state prediction with calibrated model

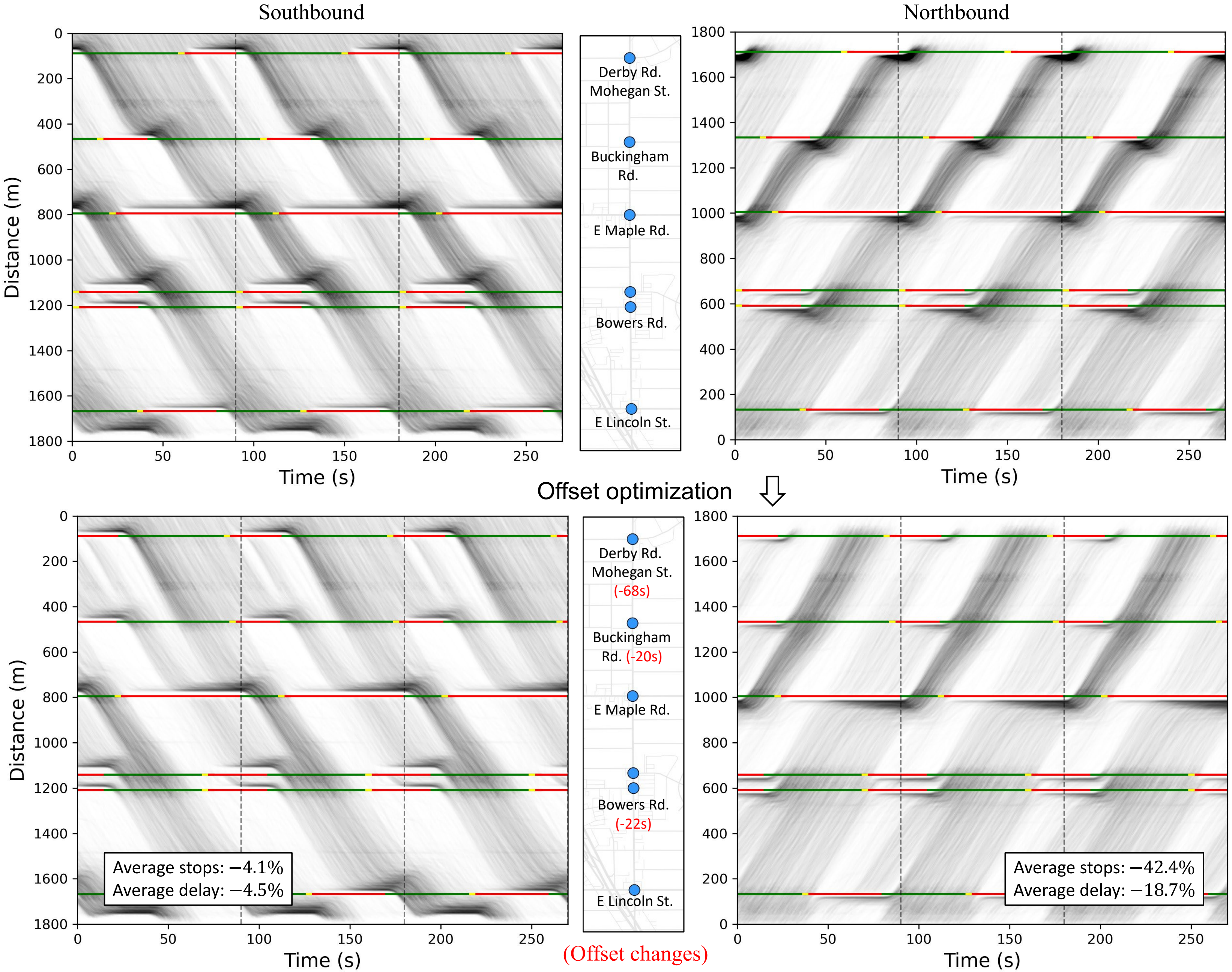


- The calibrated traffic flow model can be used to predict what will happen if the signal timing plan is changed
- In this way, we can design optimization program (e.g., Bayesian optimization) to get the optimal traffic signal parameters

Before-and-after comparison for Adams Rd.

Before optimization
(2022 March 9th -27th ,
Weekdays, Mid-day)

After optimization
(2022 April 4th -22nd ,
Weekdays, Mid-day)



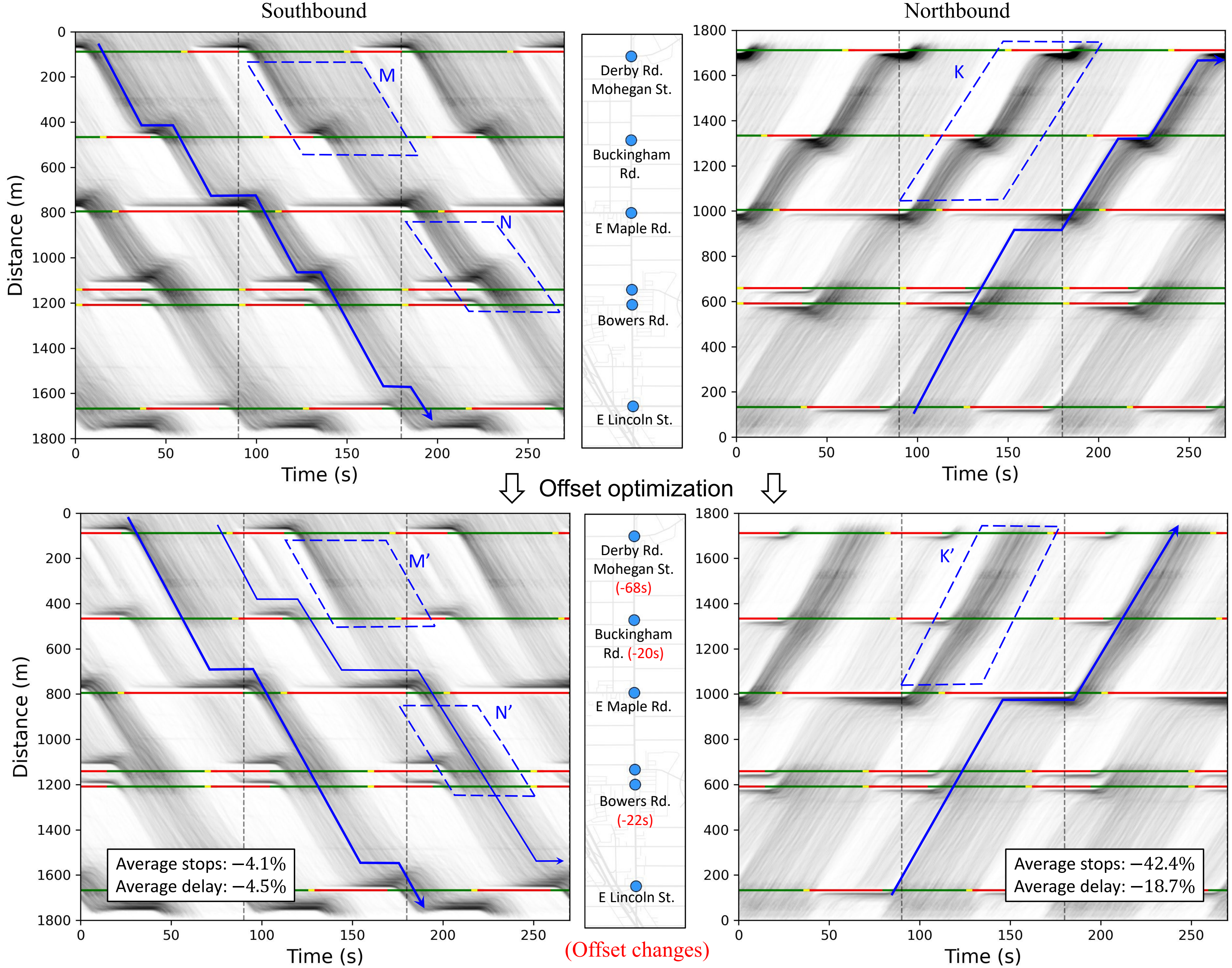
Before-and-after comparison for Adams Rd.

Before optimization

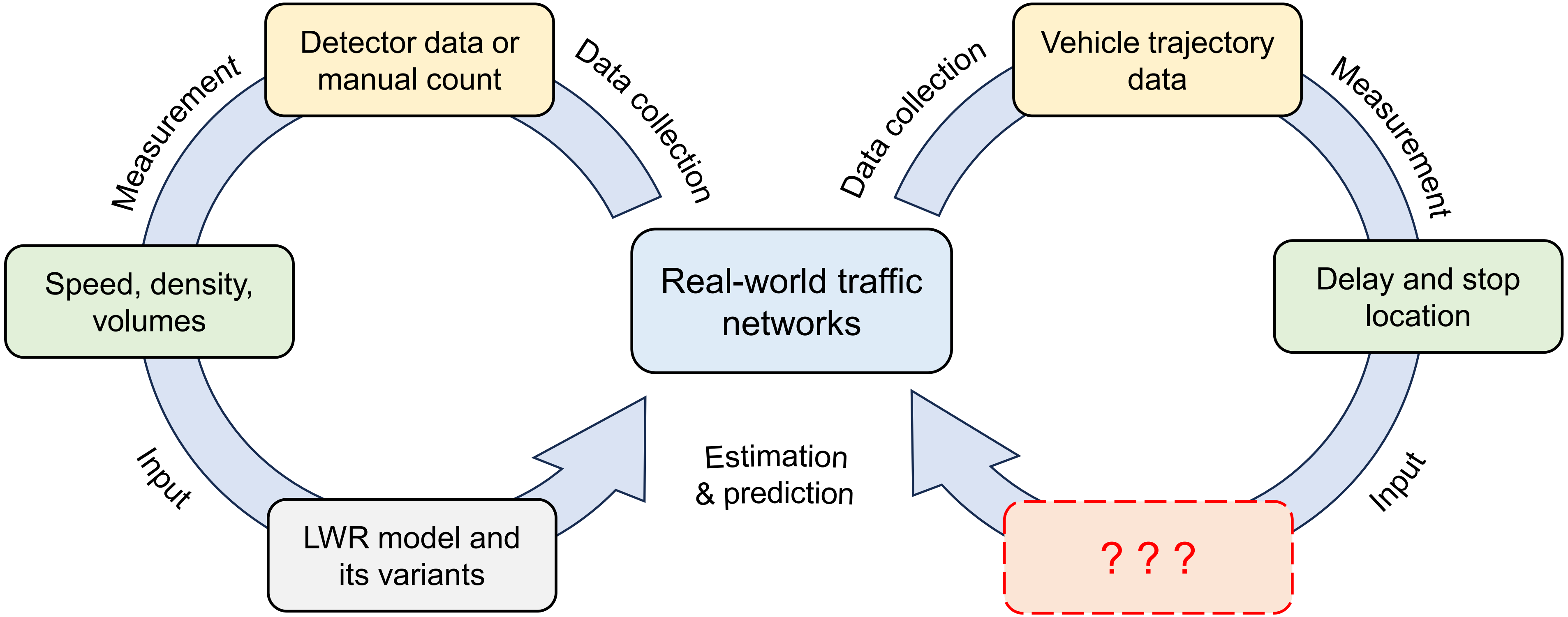
(2022 March 9th -27th , Weekdays, Mid-day)

After optimization

(2022 April 4th -22nd , Weekdays, Mid-day)



Probabilistic time-space model



Detector data or manual count

Data collection

Measurement

Speed, density, volumes

Input

LWR model and its variants

Real-world traffic networks

Estimation & prediction

Vehicle trajectory data

Data collection

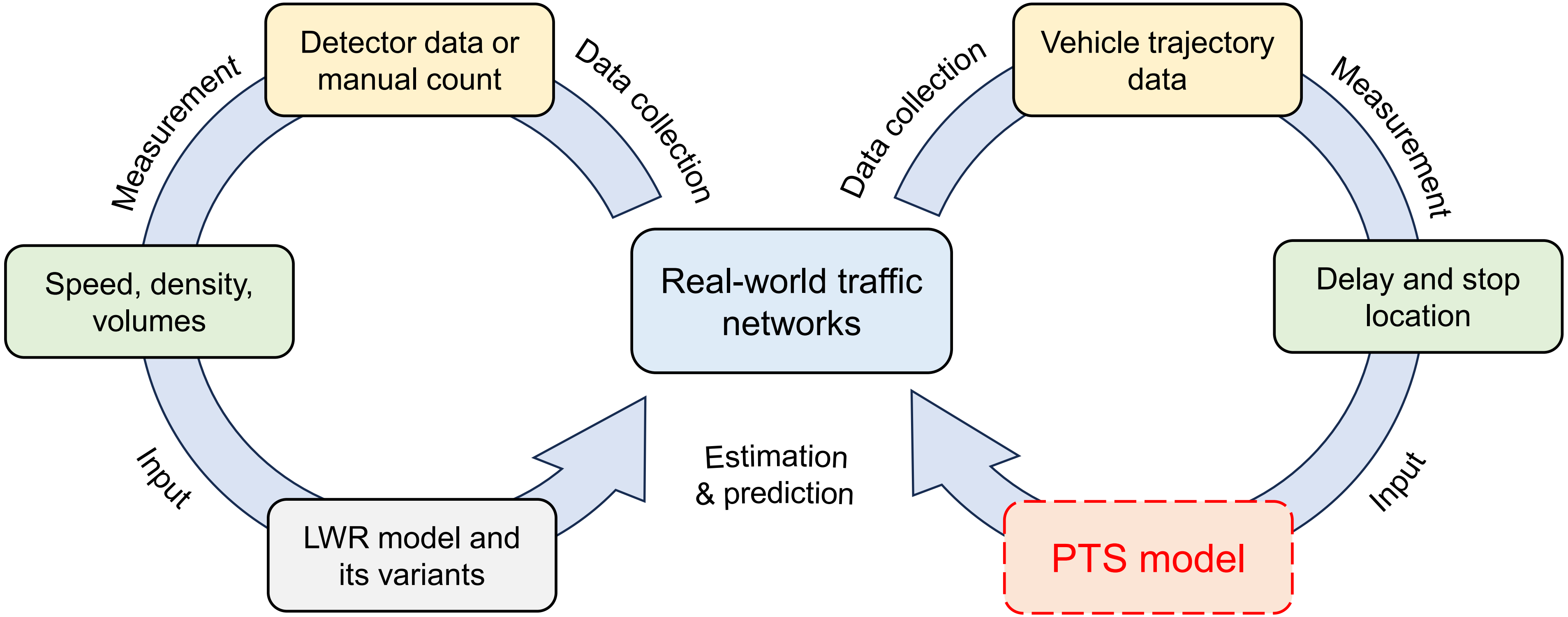
Measurement

Delay and stop location

Input

? ? ?

Probabilistic time-space model



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PTS model

Reading

- Wang, Xingmin, Zachary Jerome, Zihao Wang, Chenhao Zhang, Shengyin Shen, Vivek Vijaya Kumar, Fan Bai et al. "Traffic light optimization with low penetration rate vehicle trajectory data." Nature communications 15, no. 1 (2024): 1306.