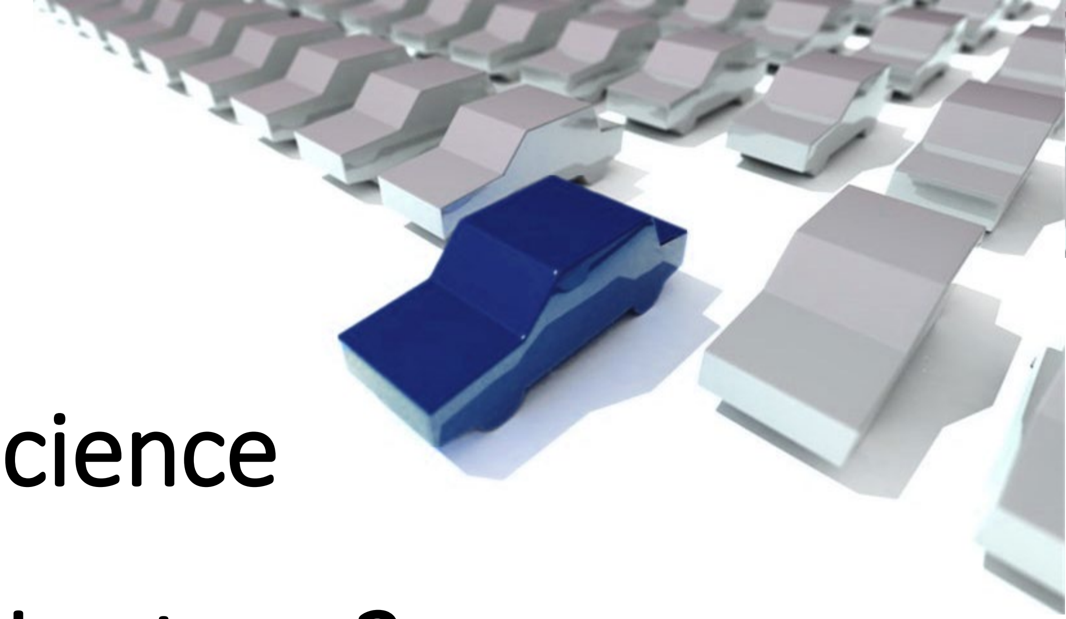

CEE 551 Traffic Science

Traffic Flow Theory Lecture 3

Shockwave solution and examples

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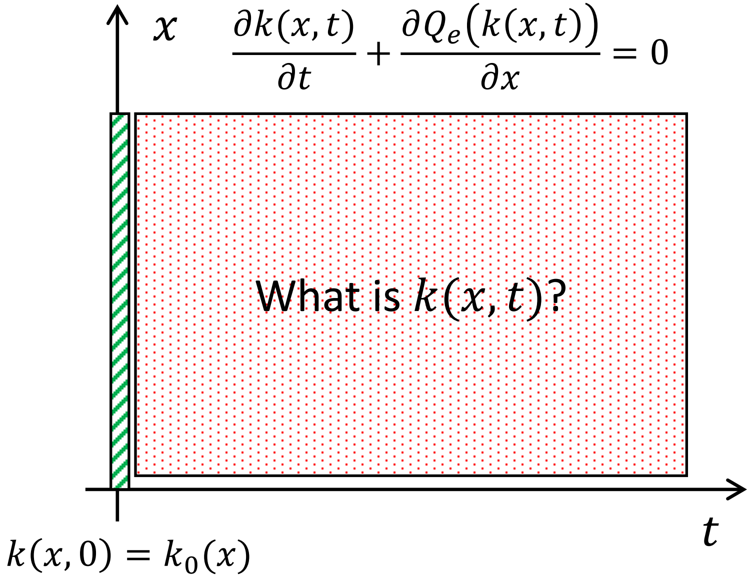
LWR model

- LWR model formulation
  - Combining the conservation law and flow-density empirical relationship, we will have the LWR (Lighthill-Whitham-Richards) model:

$$\left.\begin{array}{c}\frac{\partial k(x,t)}{\partial t}+\frac{\partial q(x,t)}{\partial x}=0\\ q(x,t)=Q_e\big(k(x,t)\big)\end{array}\right\}\ \frac{\partial k(x,t)}{\partial t}+\frac{\partial Q_e\big(k(x,t)\big)}{\partial x}=0$$

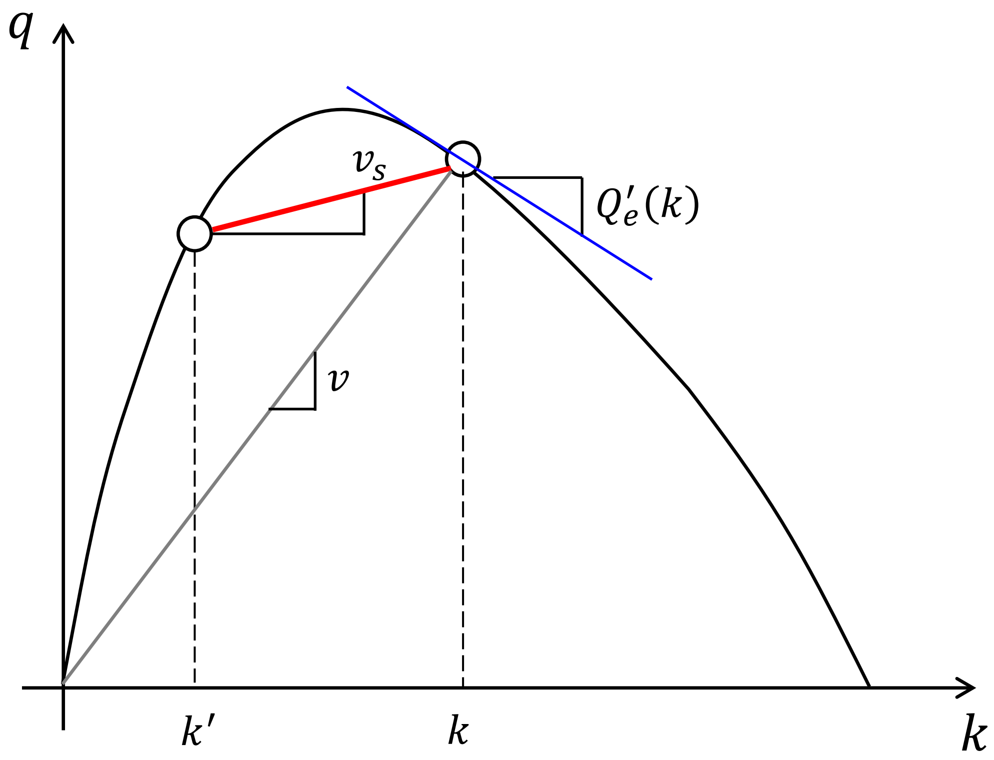
  - Initial state (boundary condition): $k(x,0)=k_0(x)$



- Solving the LWR model
  - Solving the PDE is to get $k(x,t)$ at any time t and location x given the location x

Vehicle speed, shockwave, and characteristic line

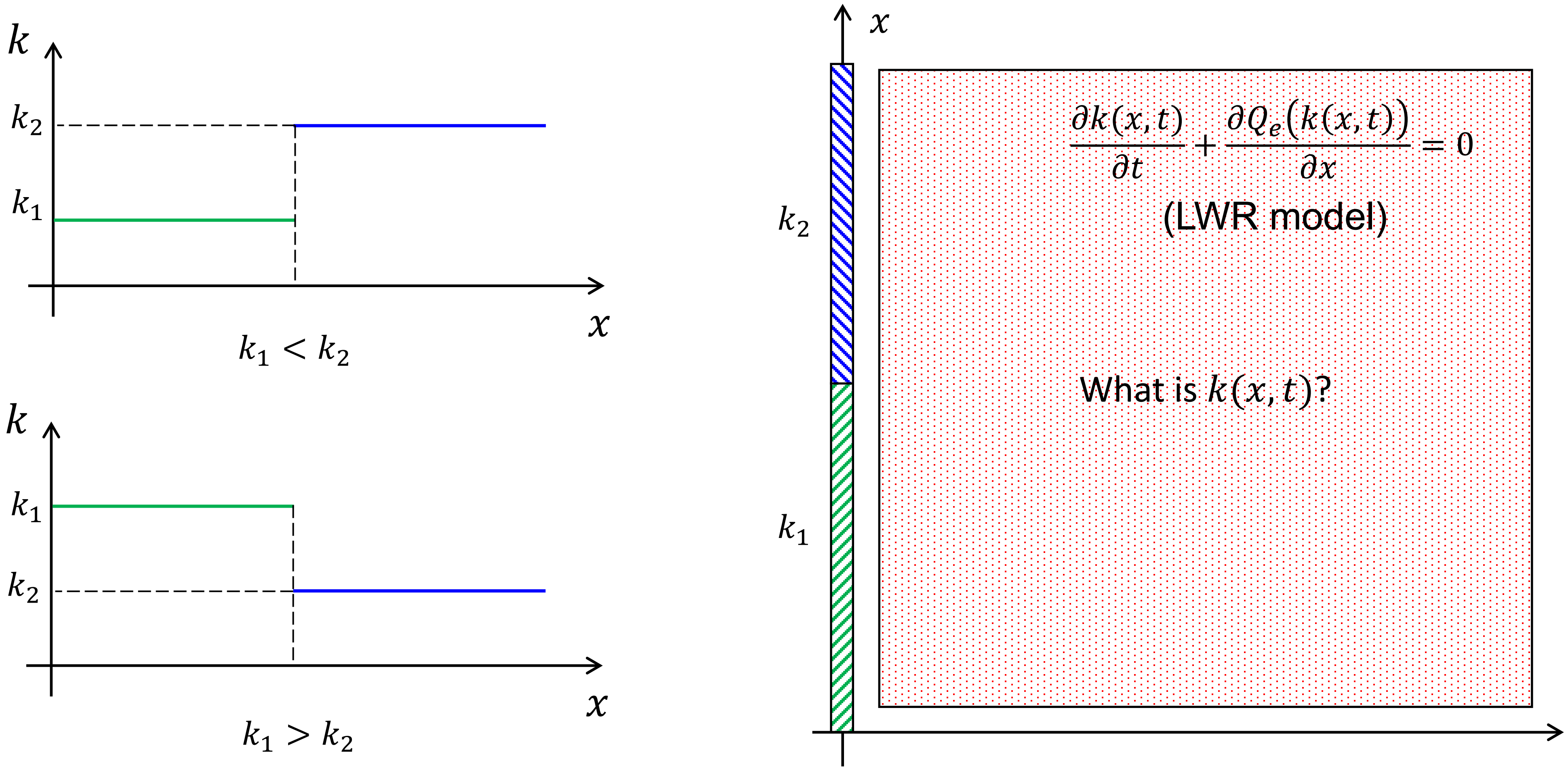
- These three have different meanings, definitions, and speeds



Name	Space-mean speed	Characteristic line	Shockwave
Meaning	Average vehicle traveling speed at certain time and location	Traffic density stays the same along the characteristic line (when solving LWR model)	Discontinuity boundary between two traffic states
Notation	$v = \frac{q}{k}$	$Q'_e(k)$	$v_s = \frac{q - q'}{k - k'}$

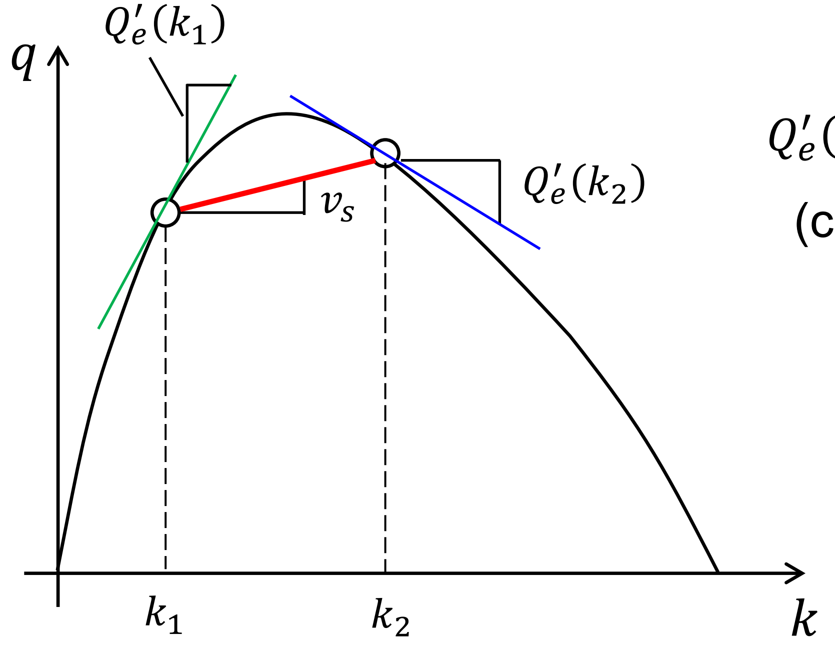
Riemann problem

- Riemann problem: solving the LWR model with step initial state



Riemann problem: shockwave solution

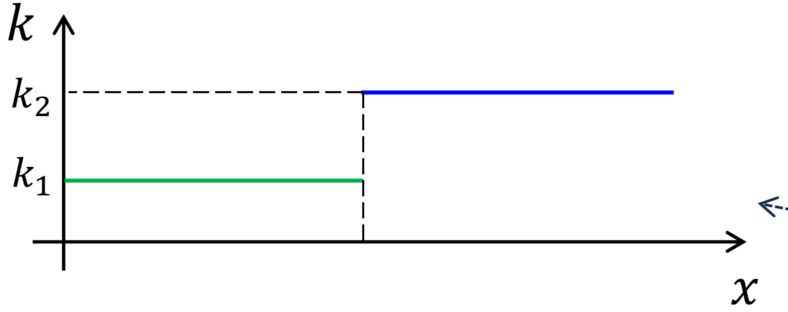
- We will get a shockwave solution if $k_1 < k_2$



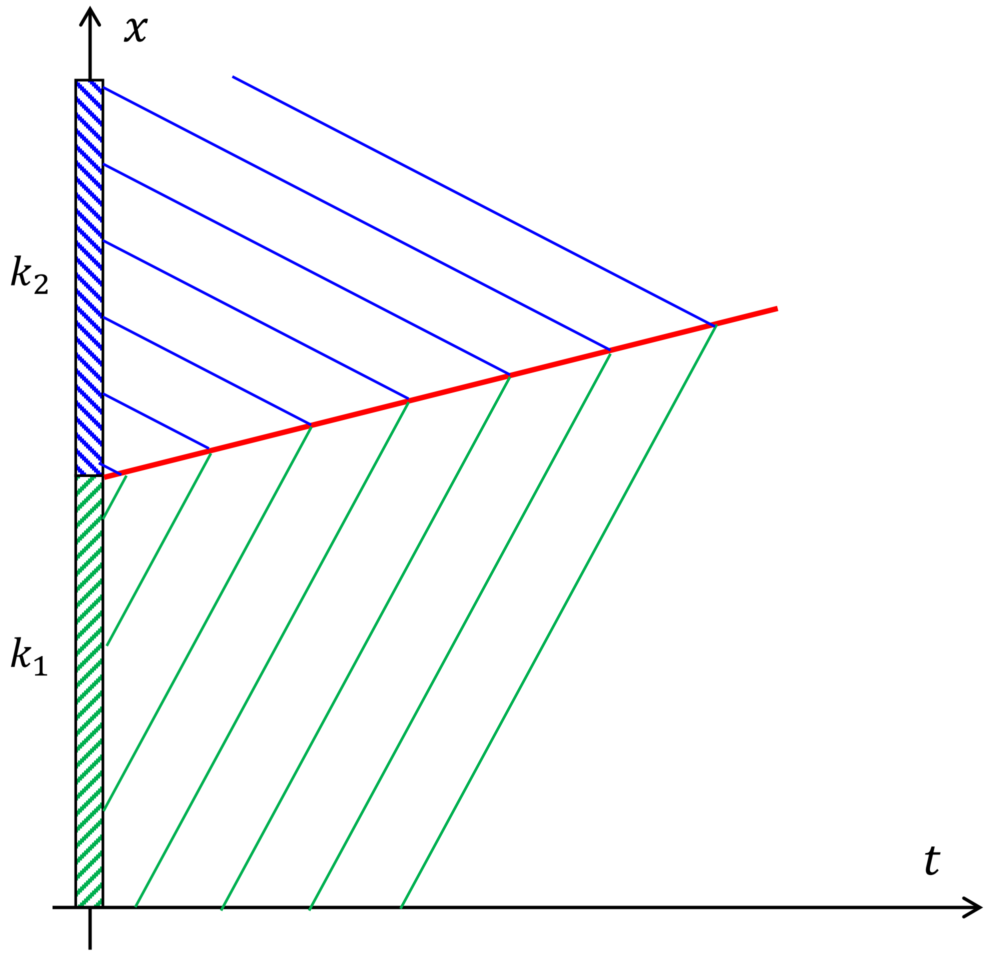
$k_1 < k_2$

$Q'_e(k_1) > Q'_e(k_1)$

(concave FD)

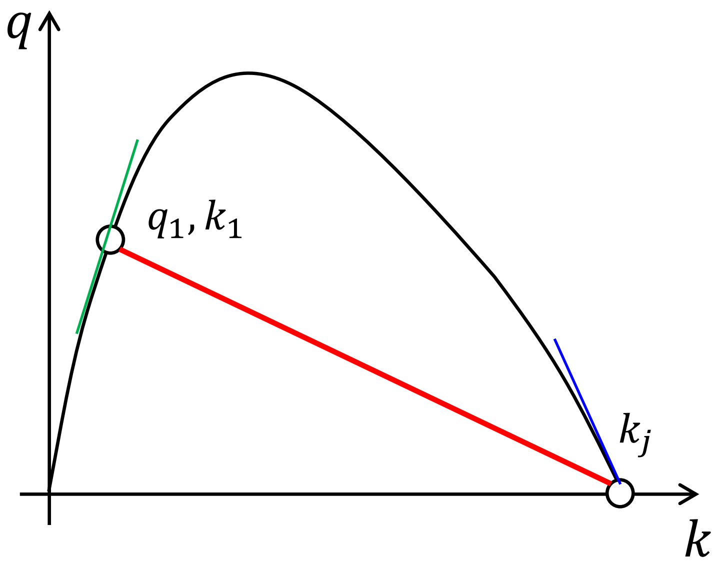


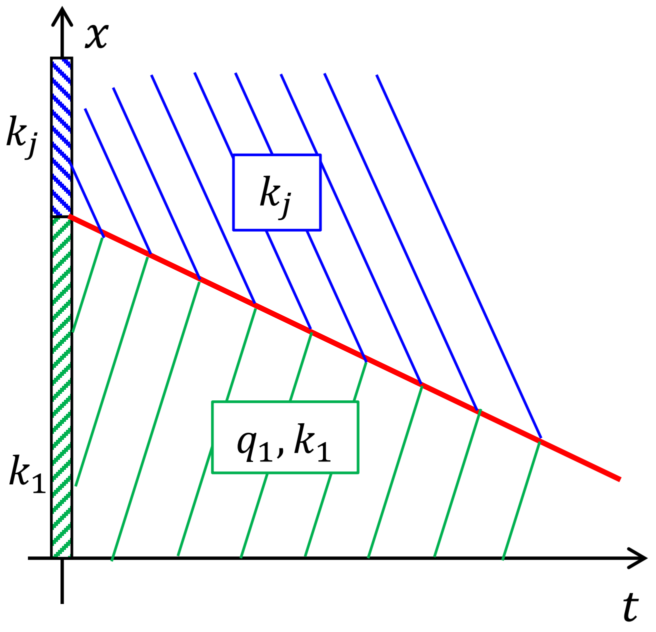
Initial state



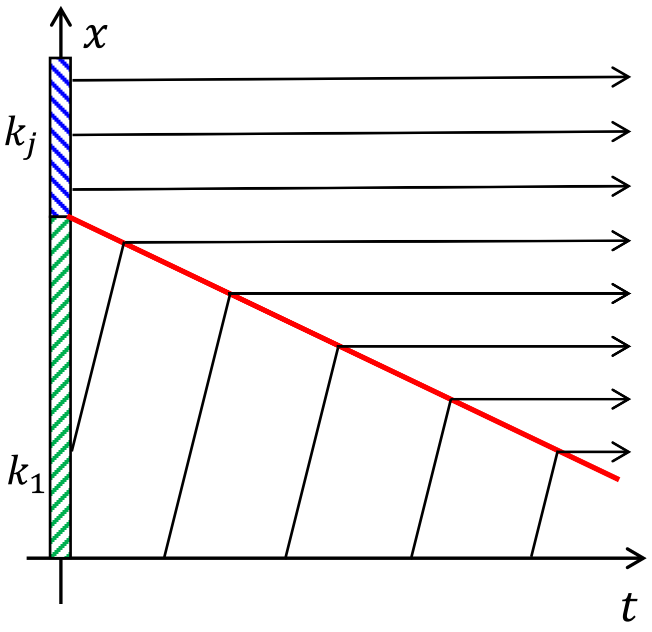
Shockwave example: queue build-up

- We have upstream constant arrival and downstream queue (jam density)

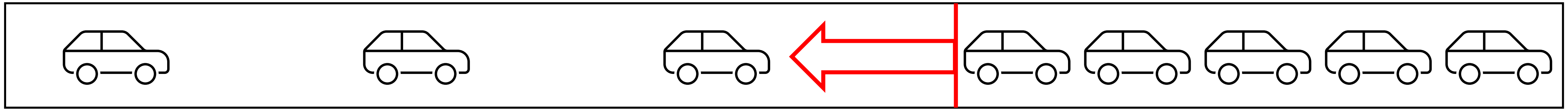




(Characteristic lines & shockwave)

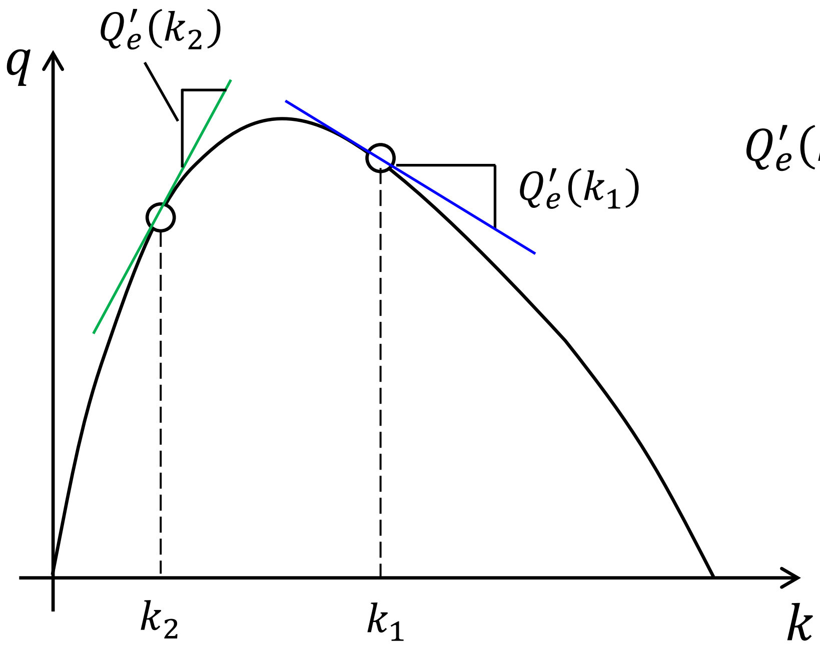


(Vehicle trajectories & shockwave)



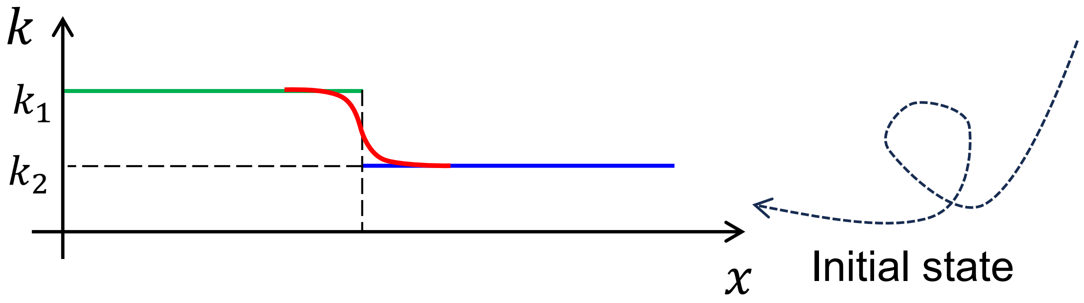
Riemann problem: rarefaction fan solution

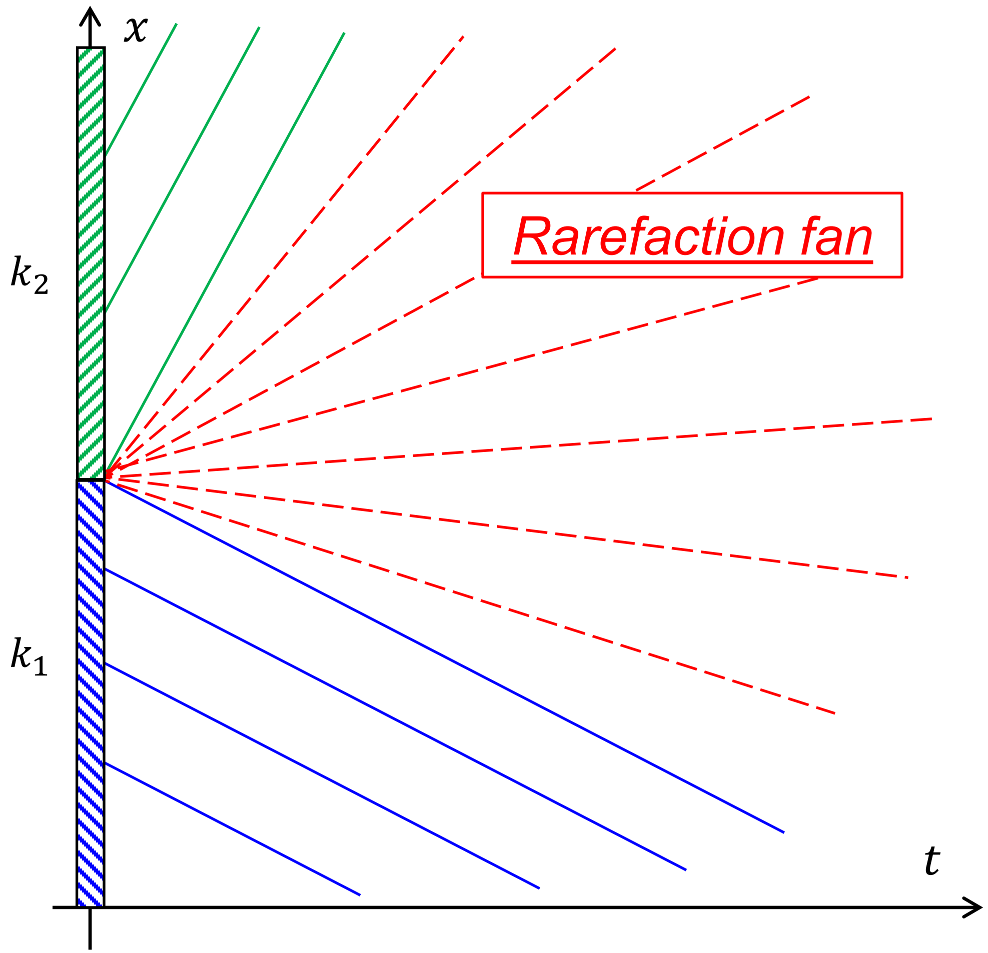
- We can twist the input step function into a continuous function



$k_1 > k_2$

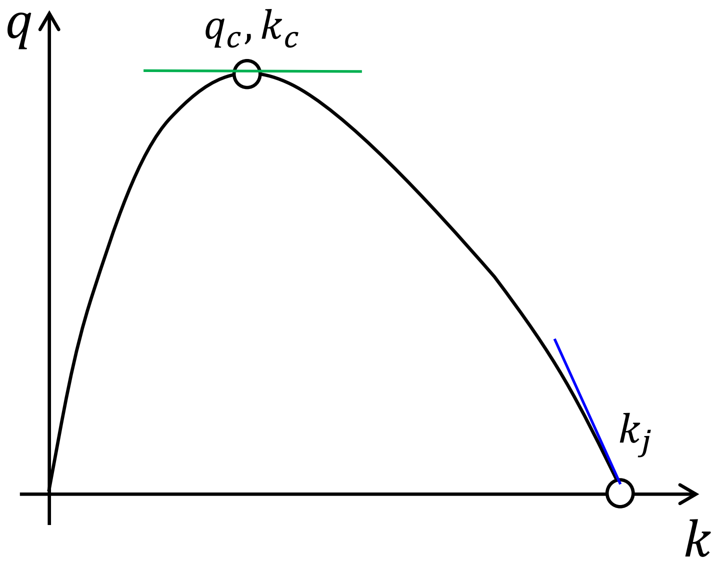
$Q'_e(k_1) < Q'_e(k_1)$

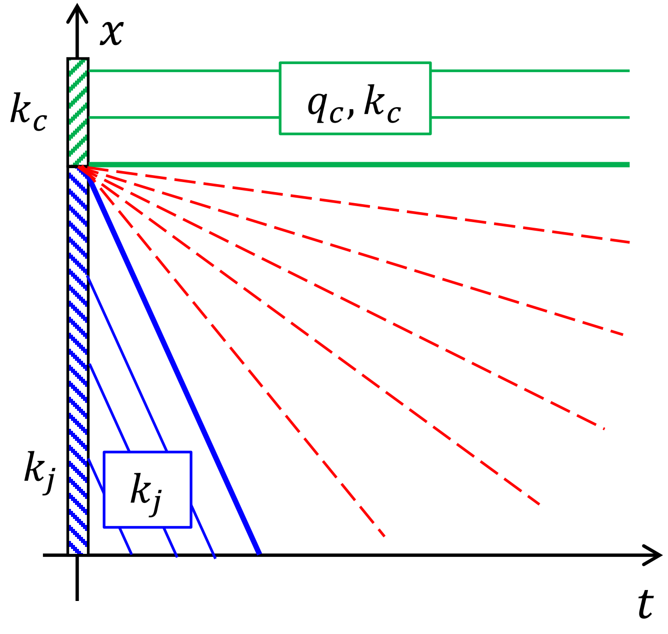




Rarefaction example: queue dissipation

- Vehicle discharging from jam density





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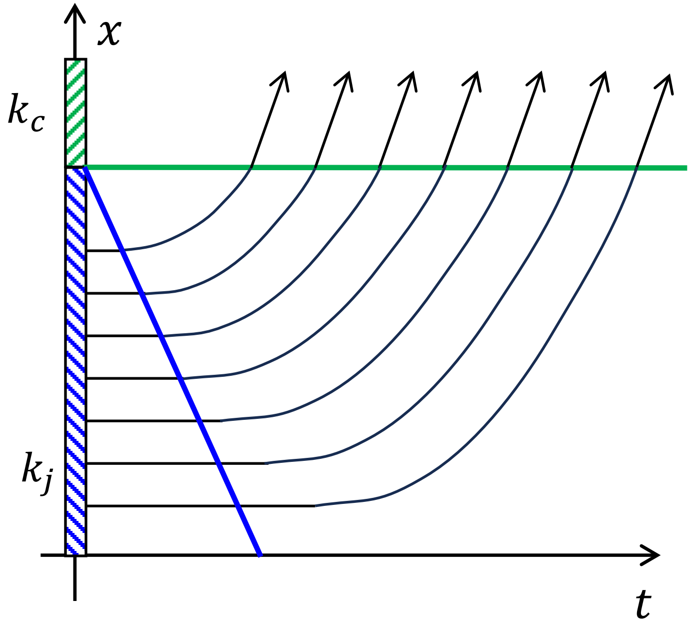

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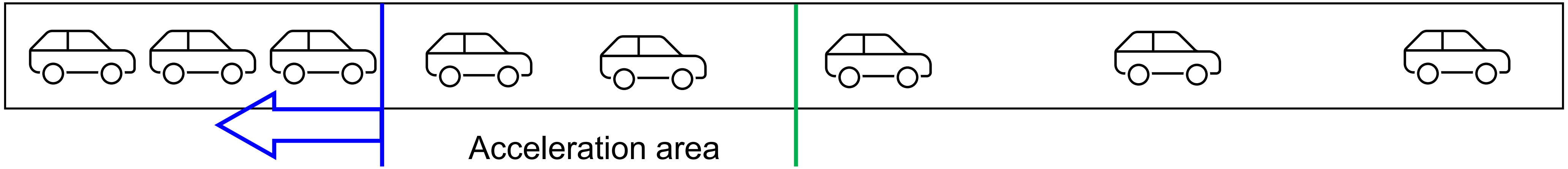

Outline

- ❑ A simplified shockwave solution
- ❑ Example 1: highway moving bottleneck
- ❑ Example 2: signalized intersection
- ❑ Calculating total delay with the TS diagram

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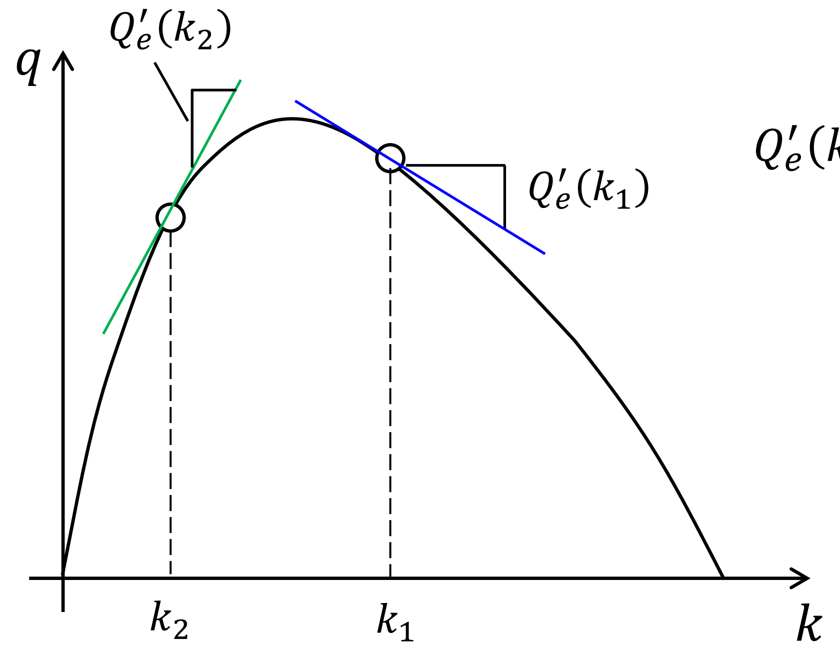
Simplified solution for rarefaction fan

- While rarefaction is a more accurate solution, we can still use “shockwave” method



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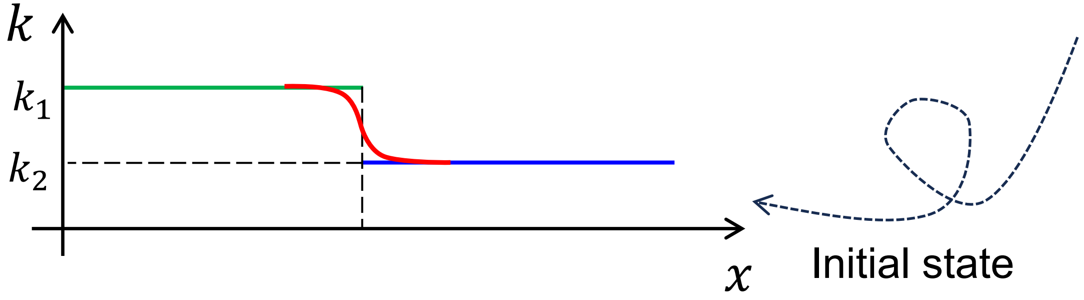
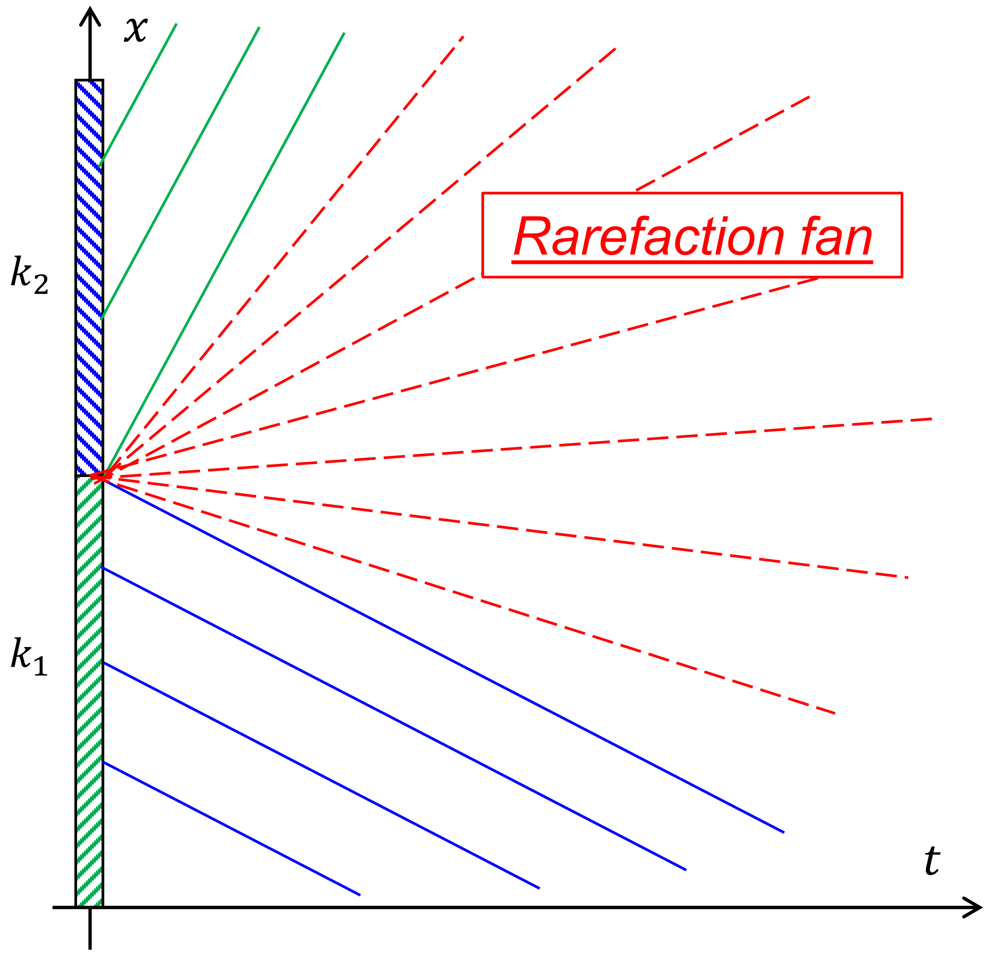
$Q'_e(k_1) < Q'_e(k_1)$





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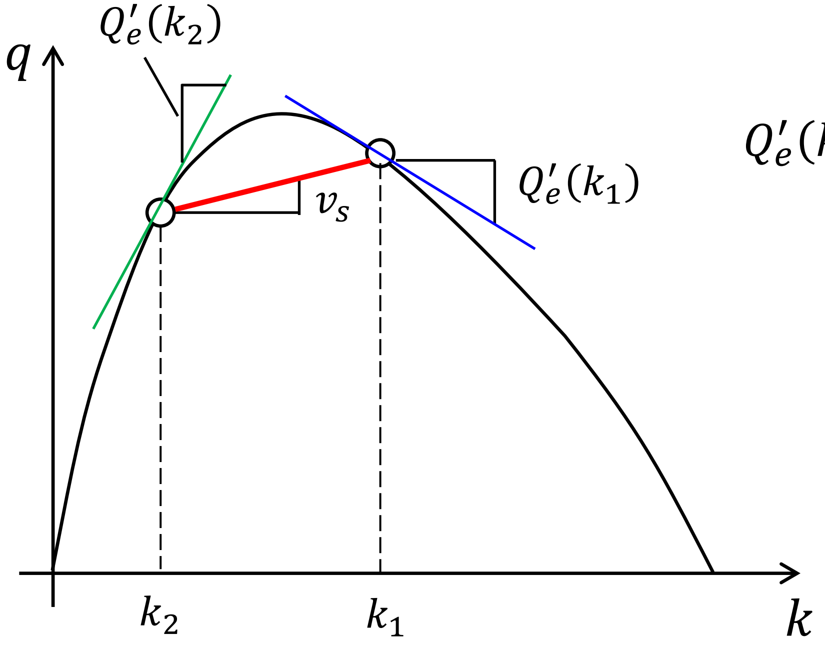
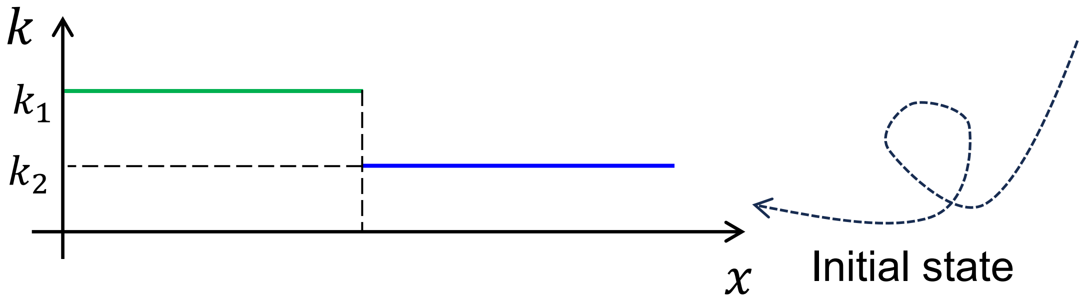
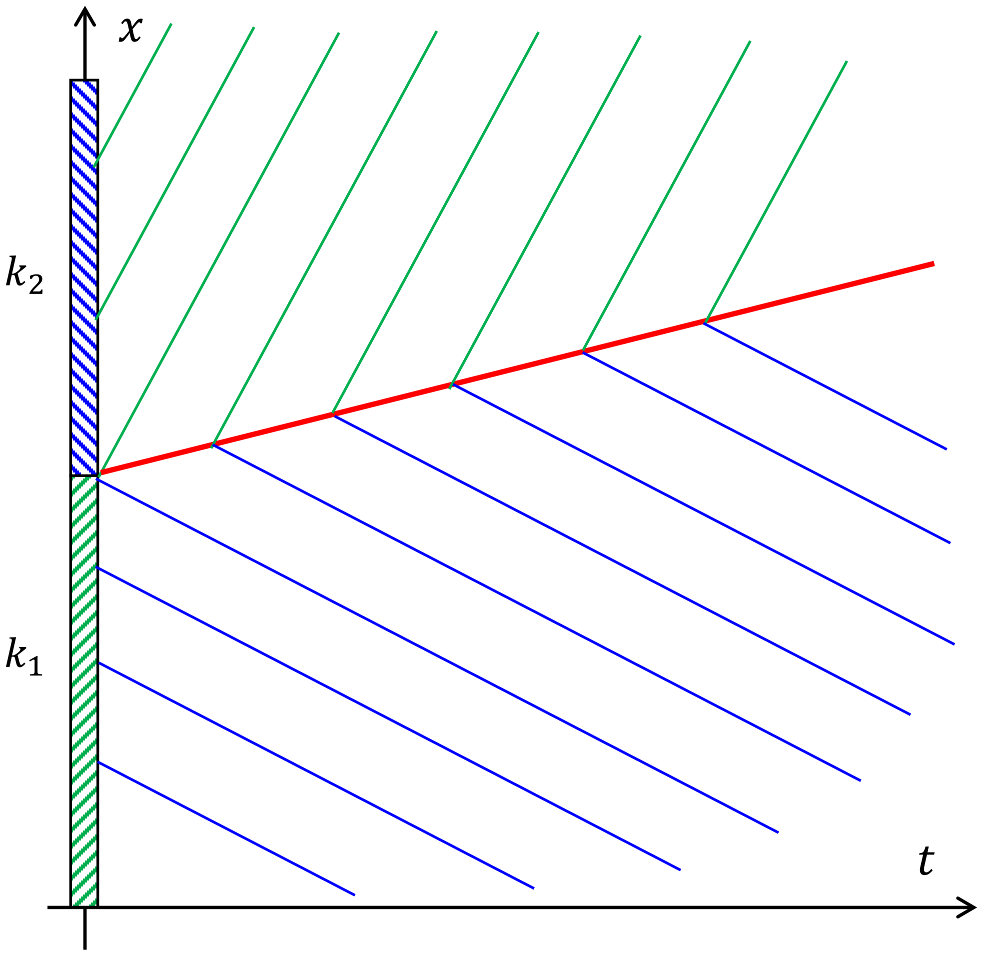


$$k_1 > k_2$$

$$Q_e'(k_1) < Q_e'(k_1)$$





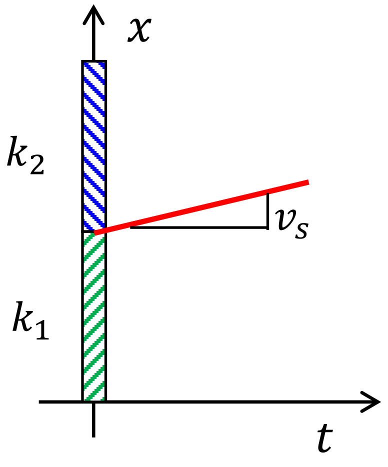
Why we can do this

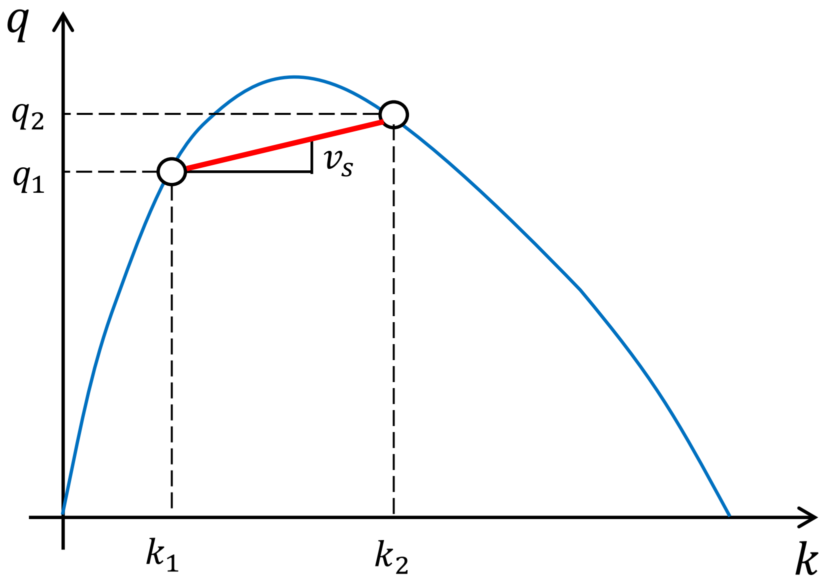
- It is easy to verify that the "shockwave solution" for the rarefaction fan also satisfies the LWR model equations (conservation law and fundamental diagram)
- Recap that when we derive the shockwave speed (based on the conservation law), we do not require the upstream density is less than the downstream density
- The "shockwave solution" for rarefaction fan essentially disregards the vehicle acceleration process

- Solution of the Riemann problem is not unique, this is why LWR model is ill-posed (solution not unique and has discontinuity)

Shockwave solution of LWR model

- If we use a shockwave method also for the rarefaction fan case, we will have a uniform "shockwave" solution for both shockwave and rarefaction fan
- Shockwave can be regarded as the boundary between stationary traffic states
- Shockwave speed is the slope of the line connecting two traffic states in FD (the derivation is based on the conservation law)





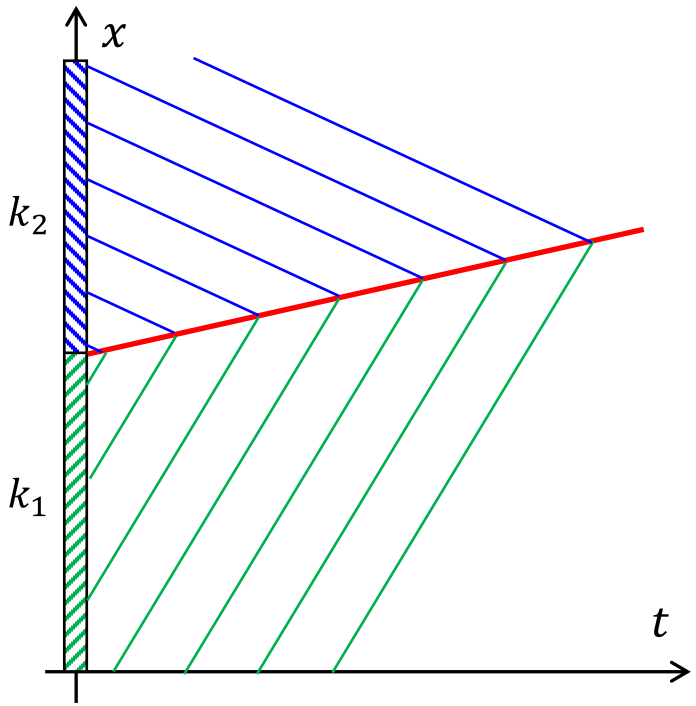
Shockwave speed

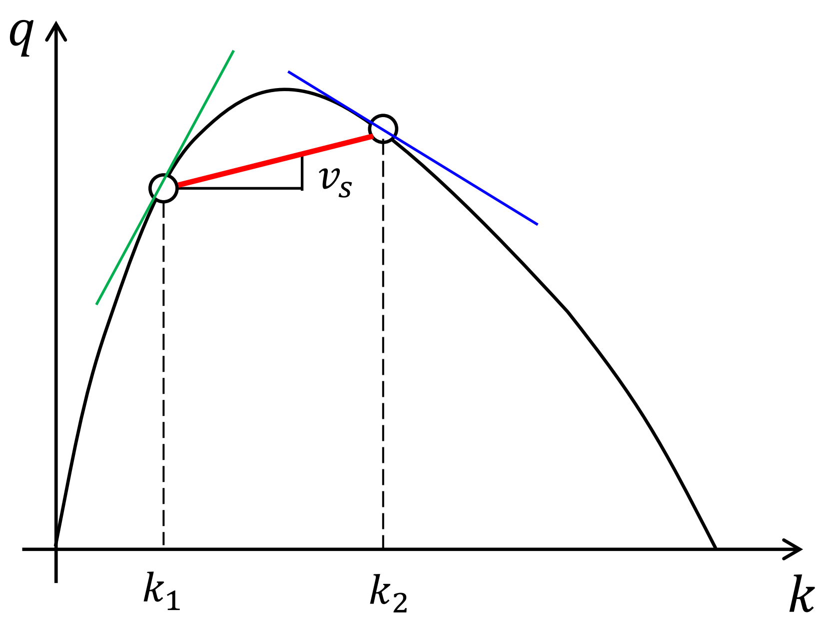
$$v_s = \frac{q_2 - q_1}{k_2 - k_1}$$

- $v_s > 0$: moving to the downstream
- $v_s = 0$: stationary
- $v_s < 0$: moving to the upstream

Summary of the shockwave theory

- Step 1: determine the traffic states on both sides and label them in FD
- Step 2: get the shockwave speed based on the given FD
- Step 3: draw the shockwave line as the boundary of the two traffic states





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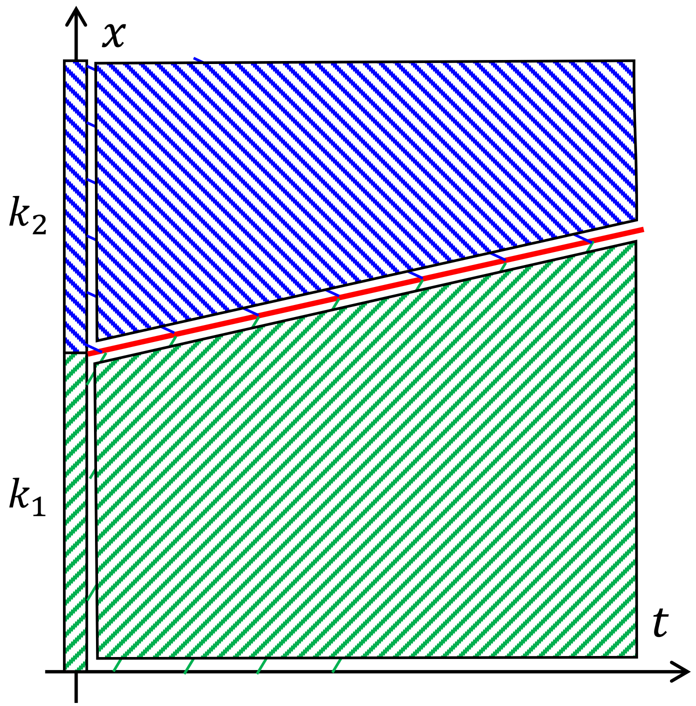

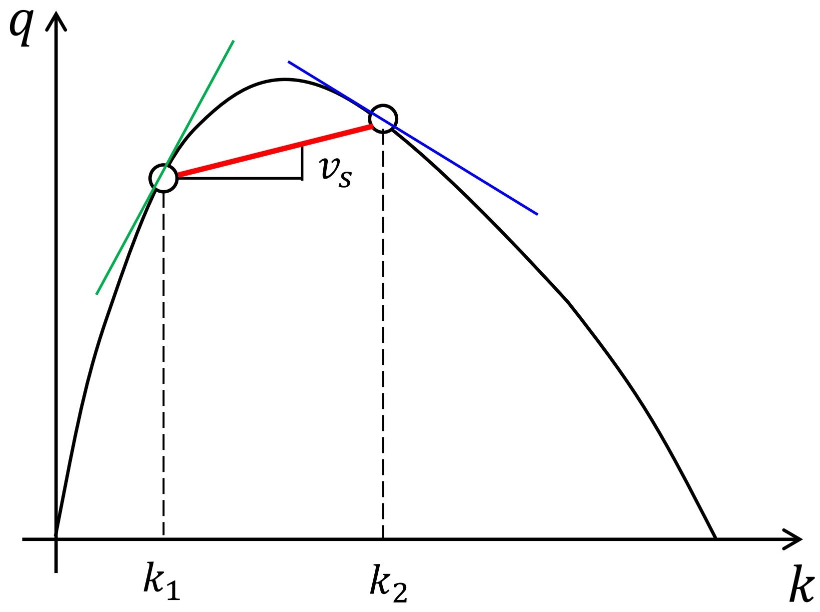

- We do not need to distinguish shockwave & rarefaction fan. They are all within the same solution framework
- We do not need to care about characteristic lines anymore, since traffic states are always uniform on both sides of the shockwave

Outline

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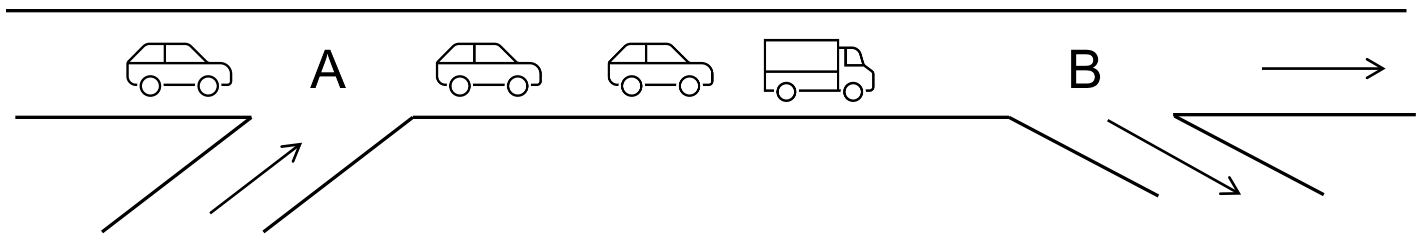
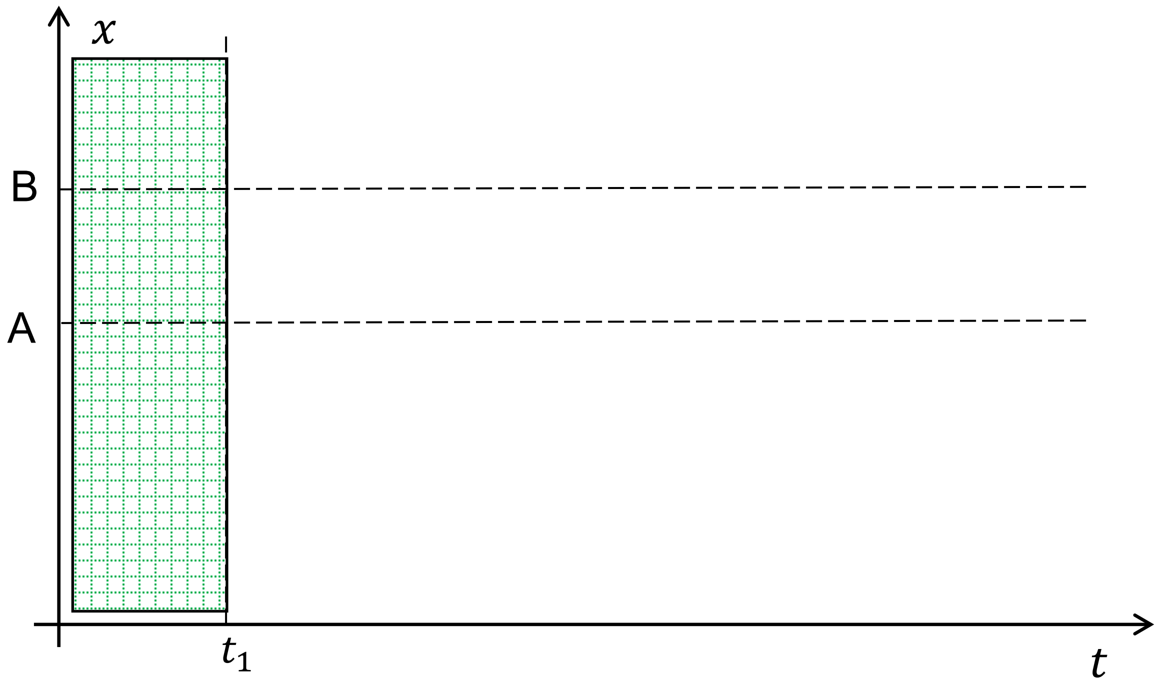
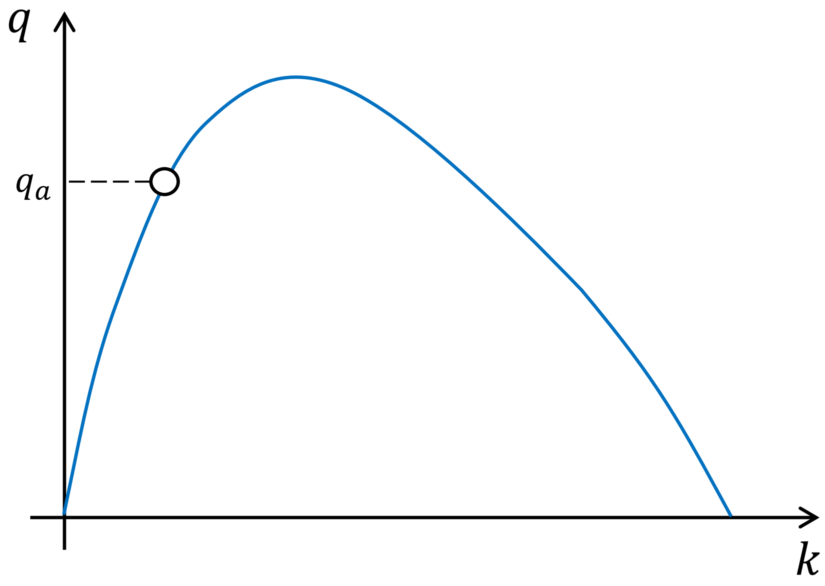
Example 1: highway moving bottleneck



A truck enters the highway at t_1 with a slower speed v_l and exits at B. Traffic volume is q_a





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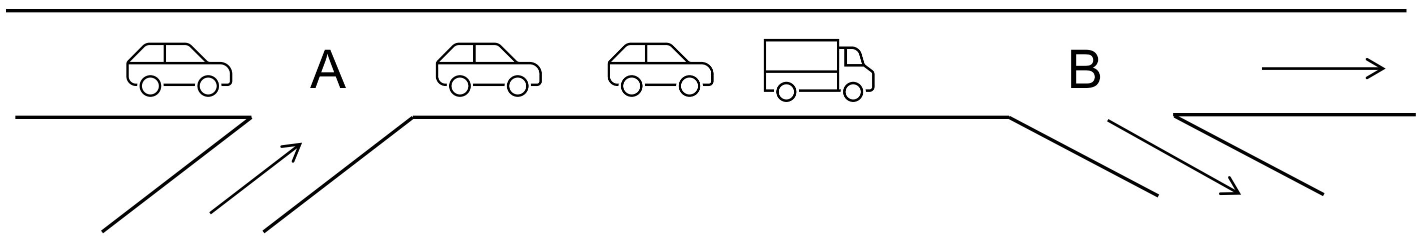

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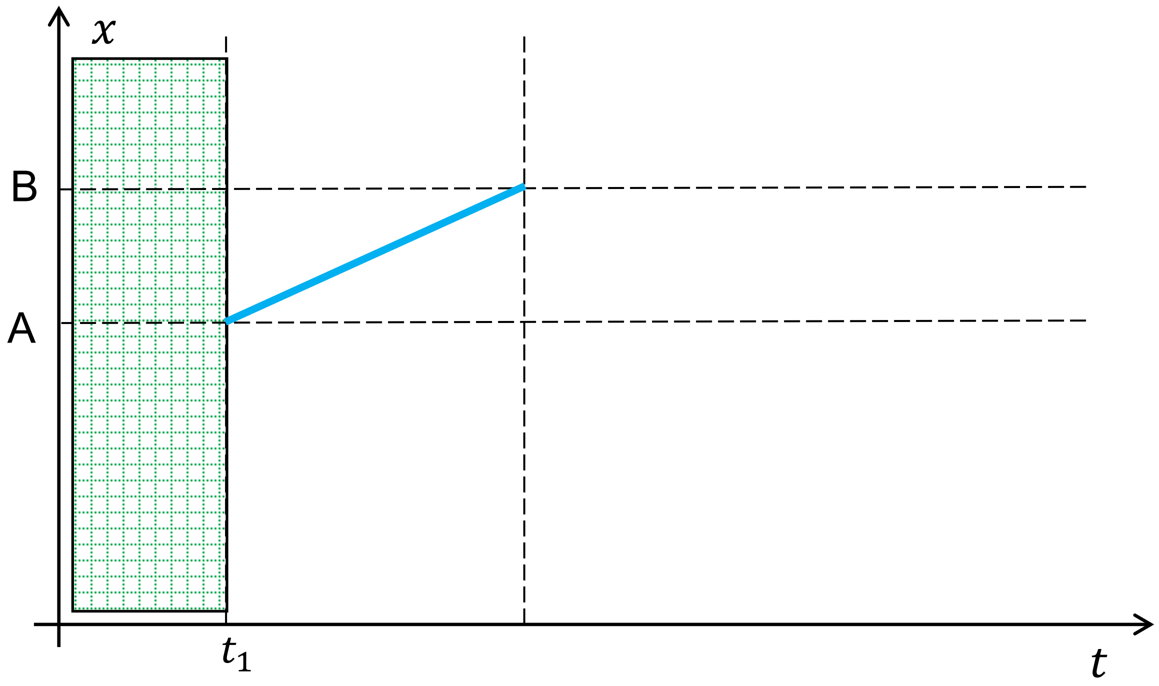

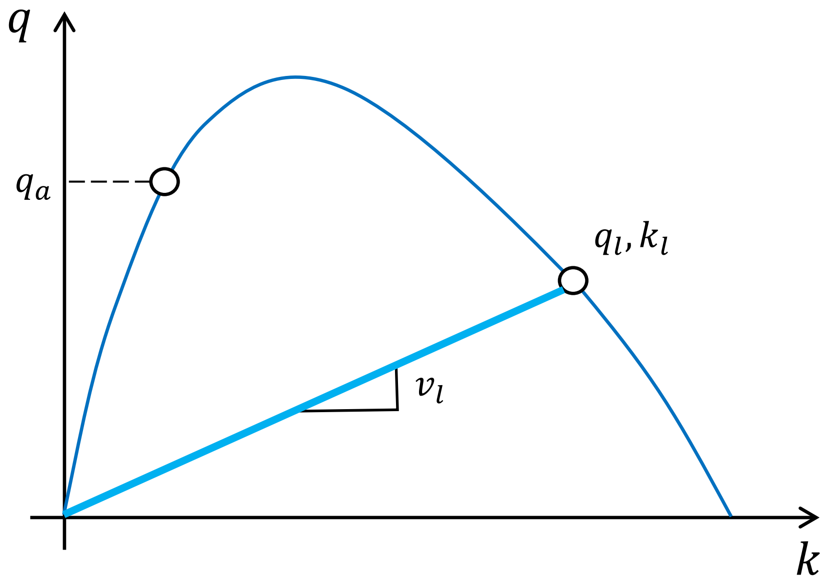

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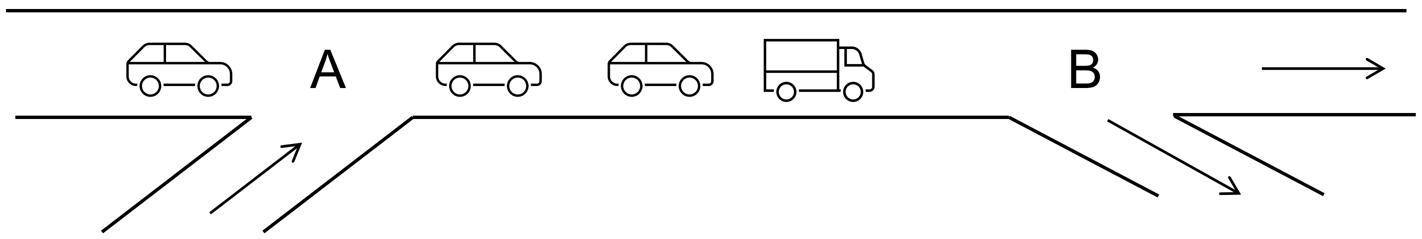

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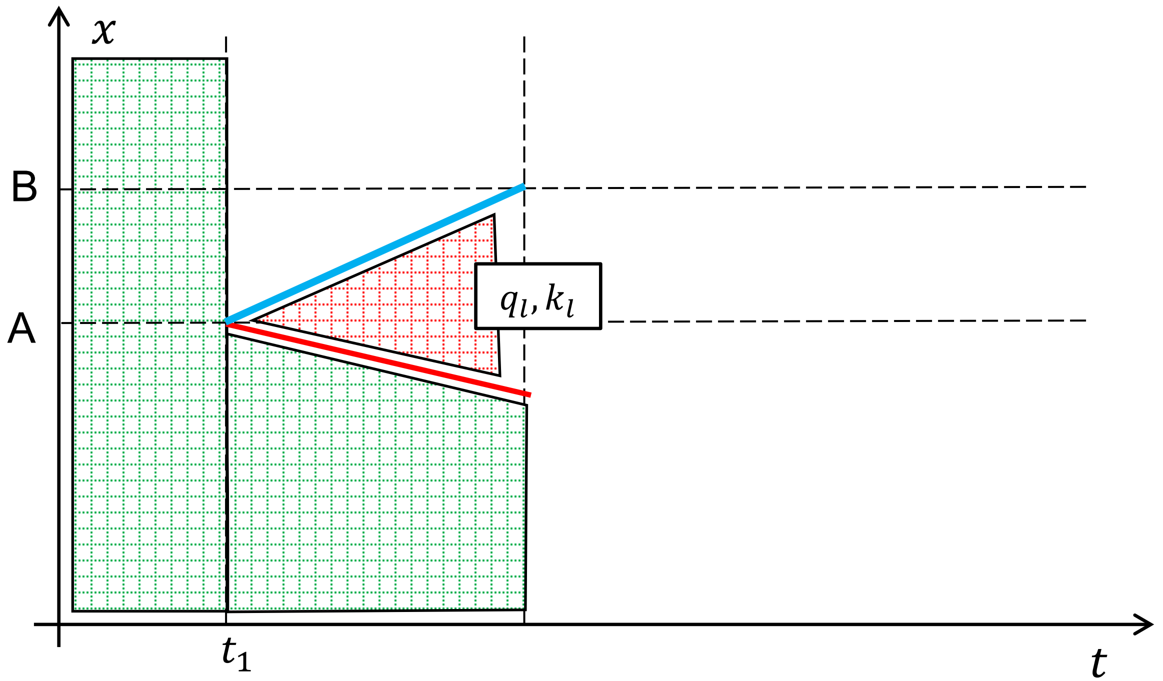

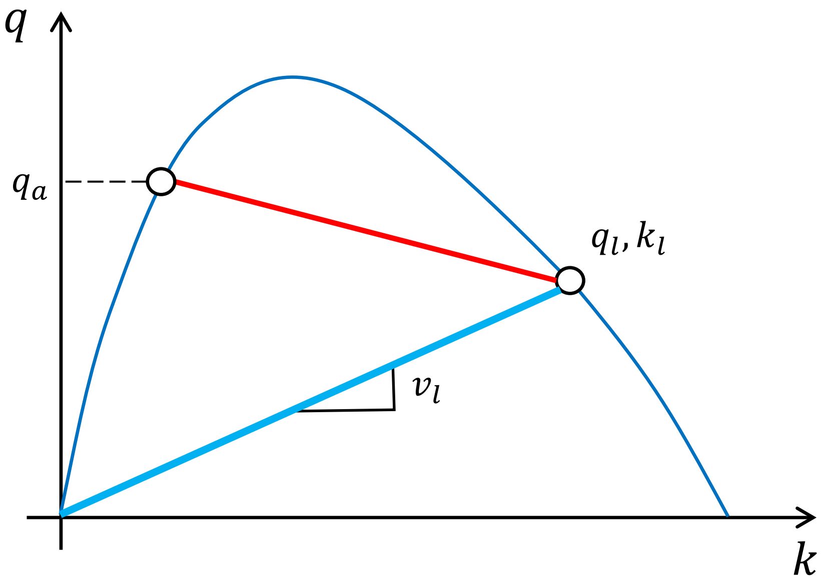

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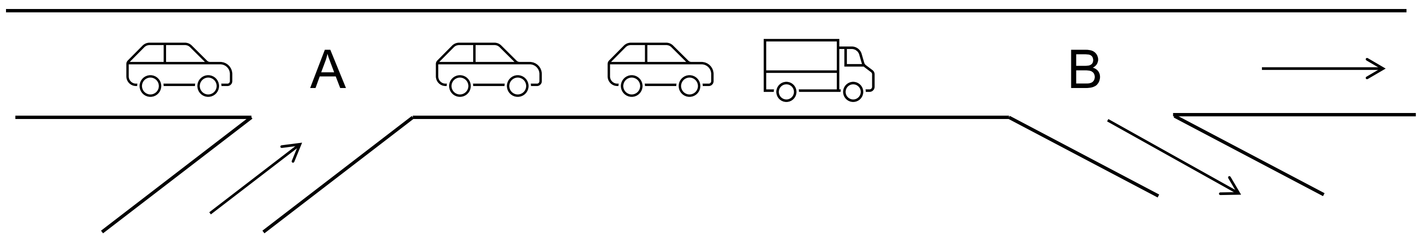

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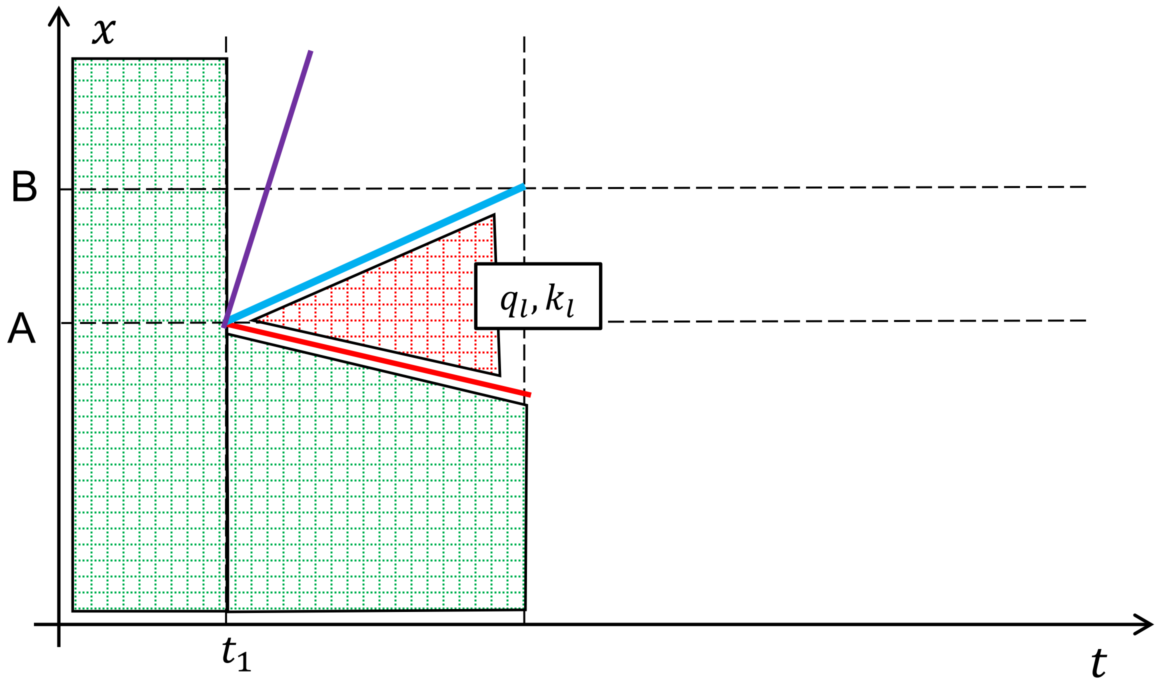

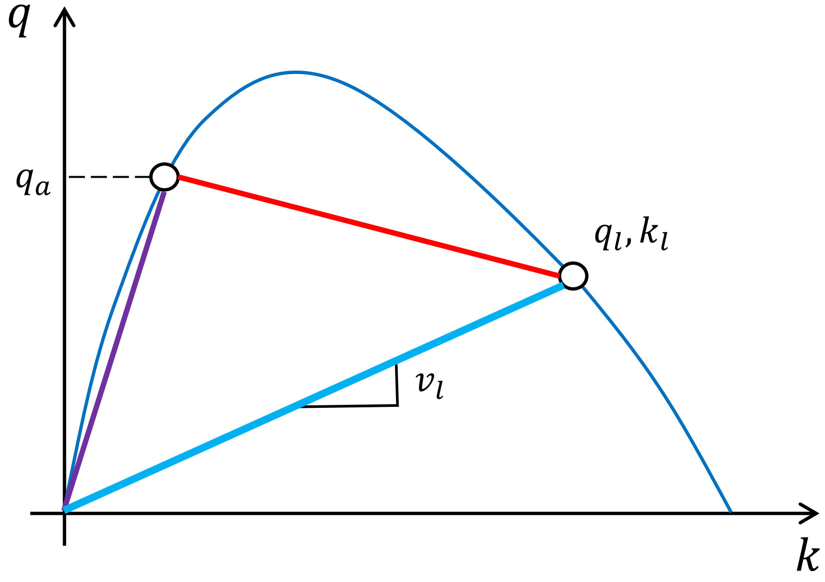

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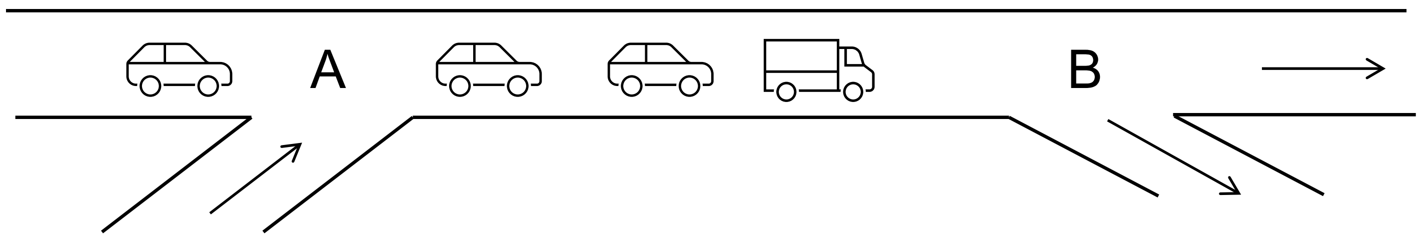

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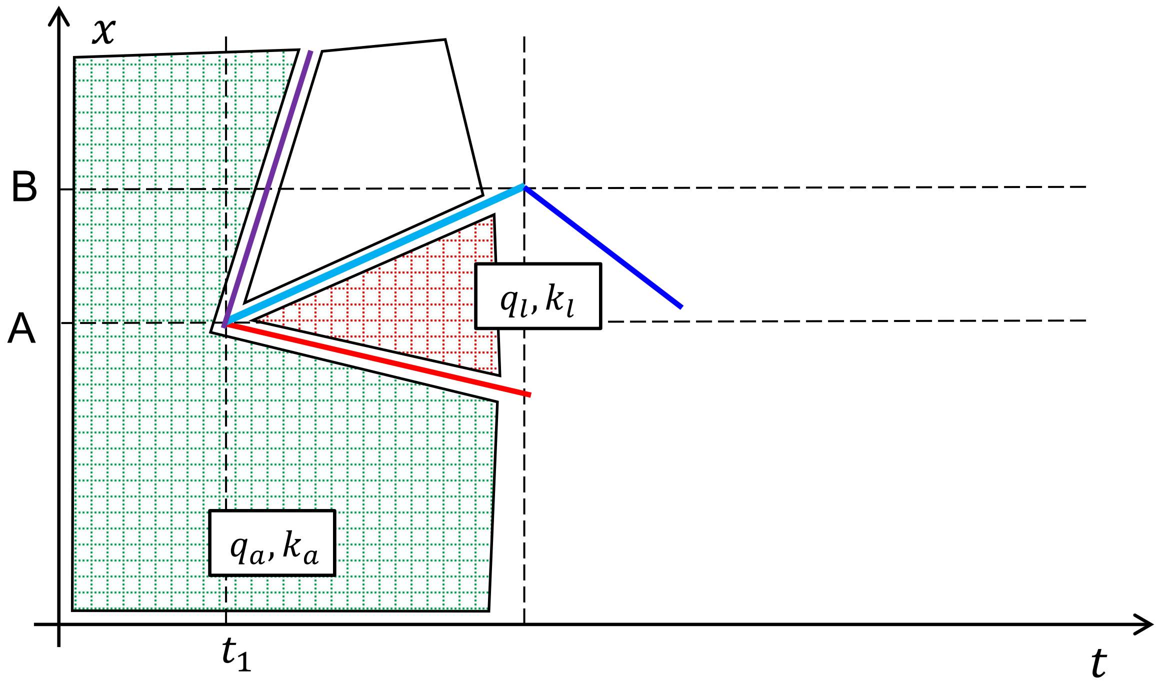

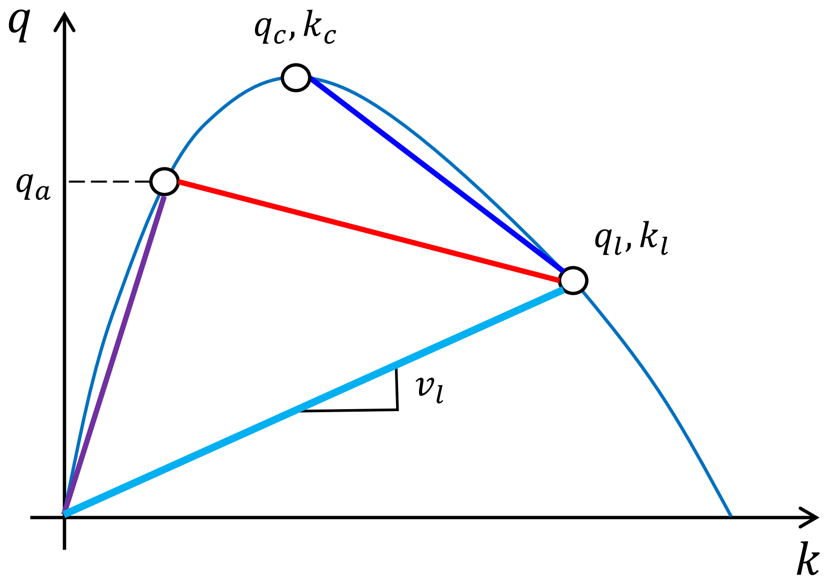

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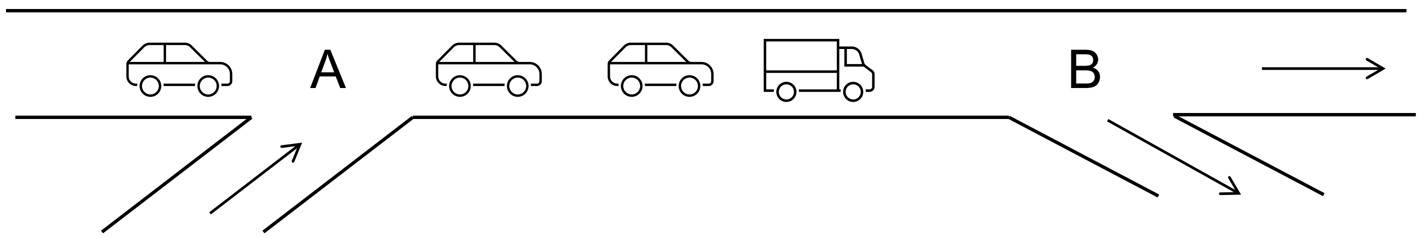

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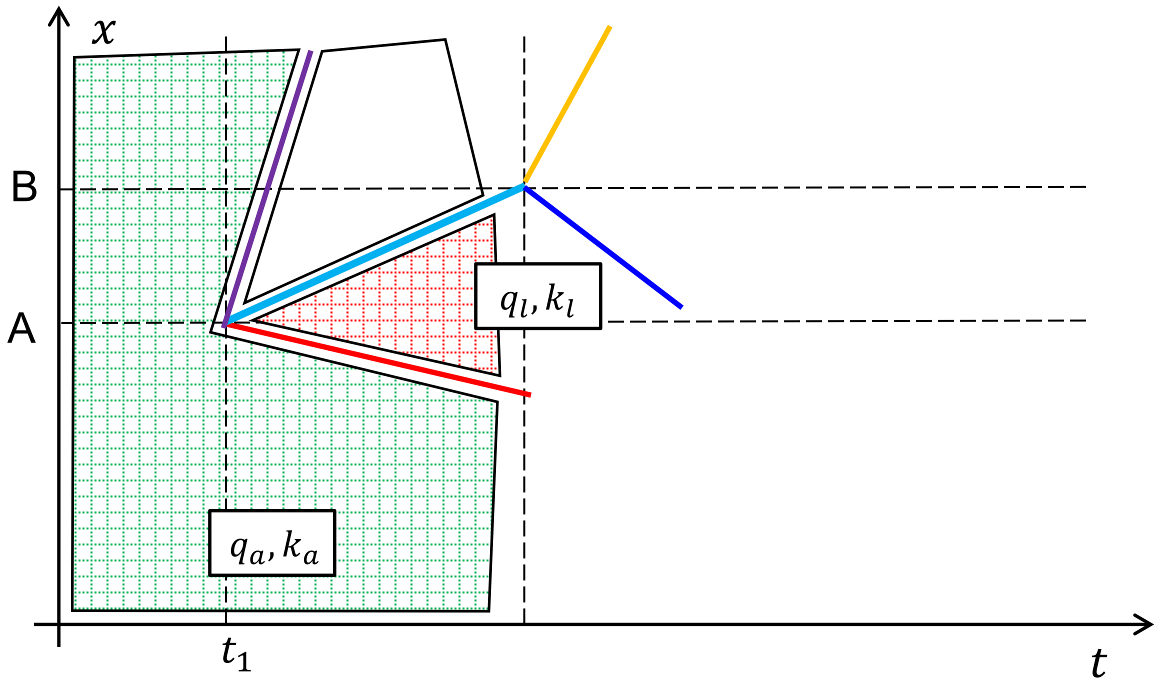

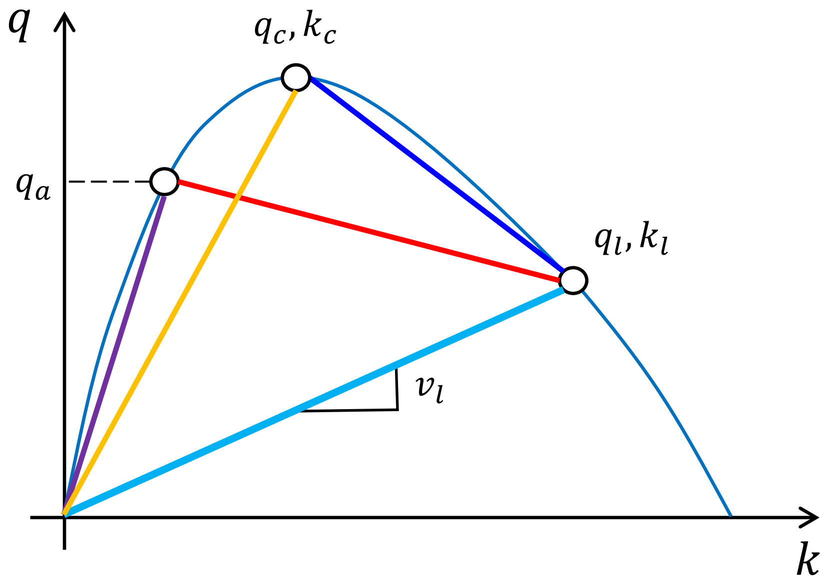

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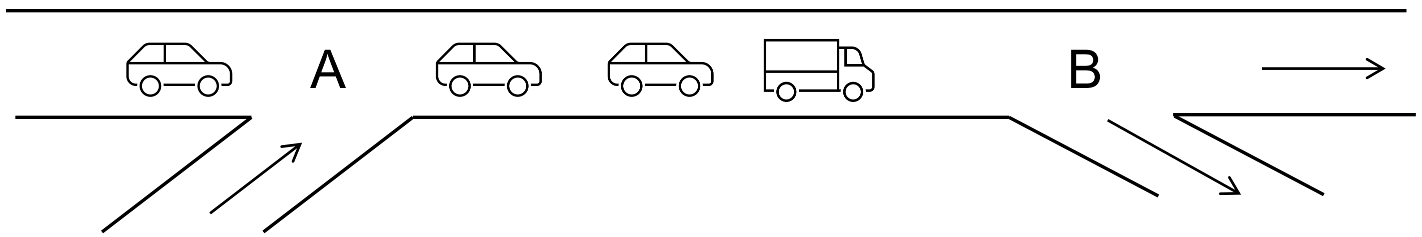

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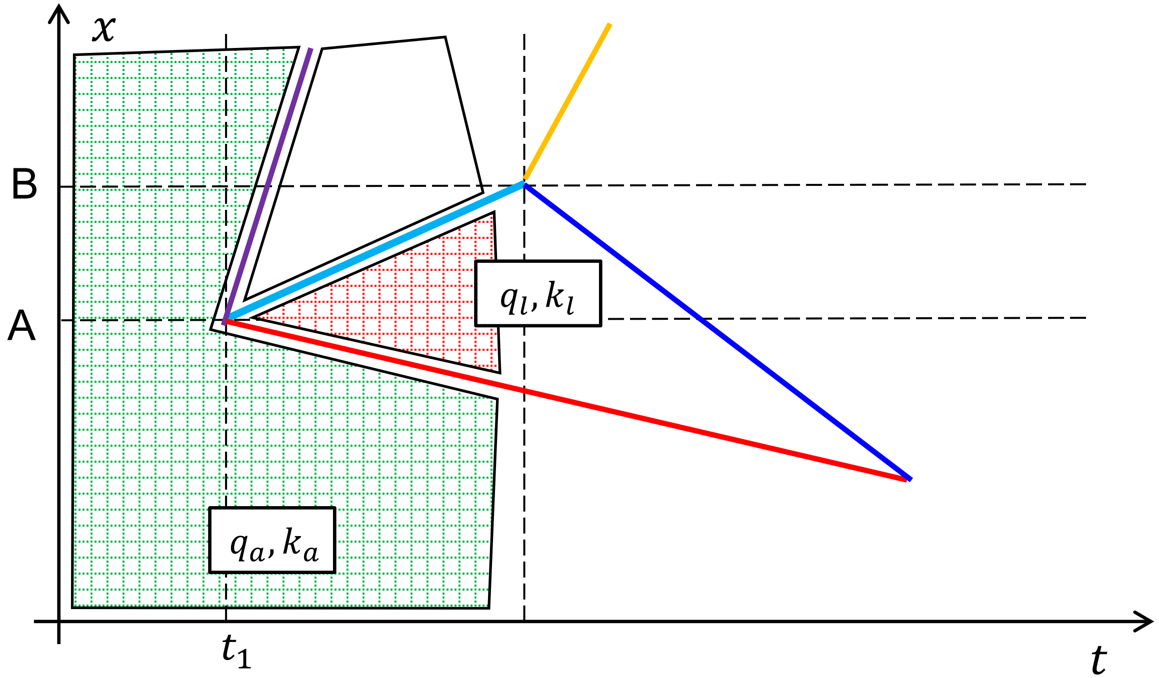

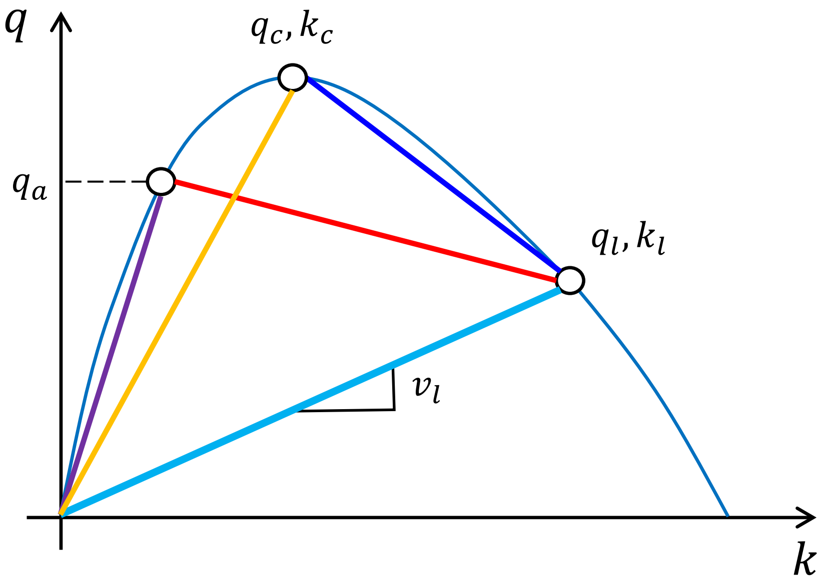

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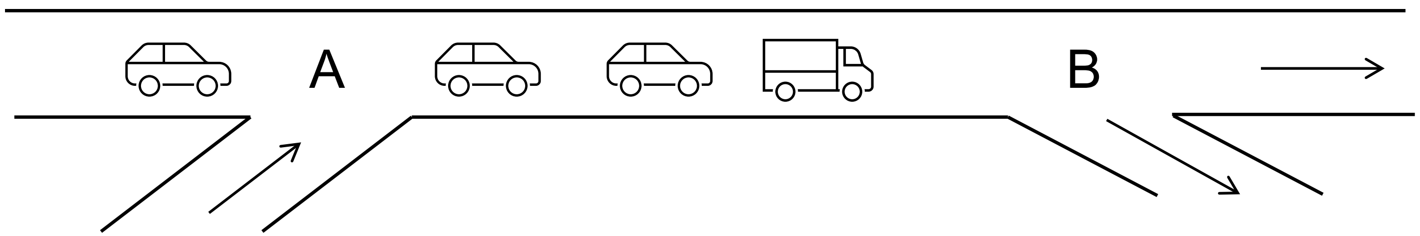

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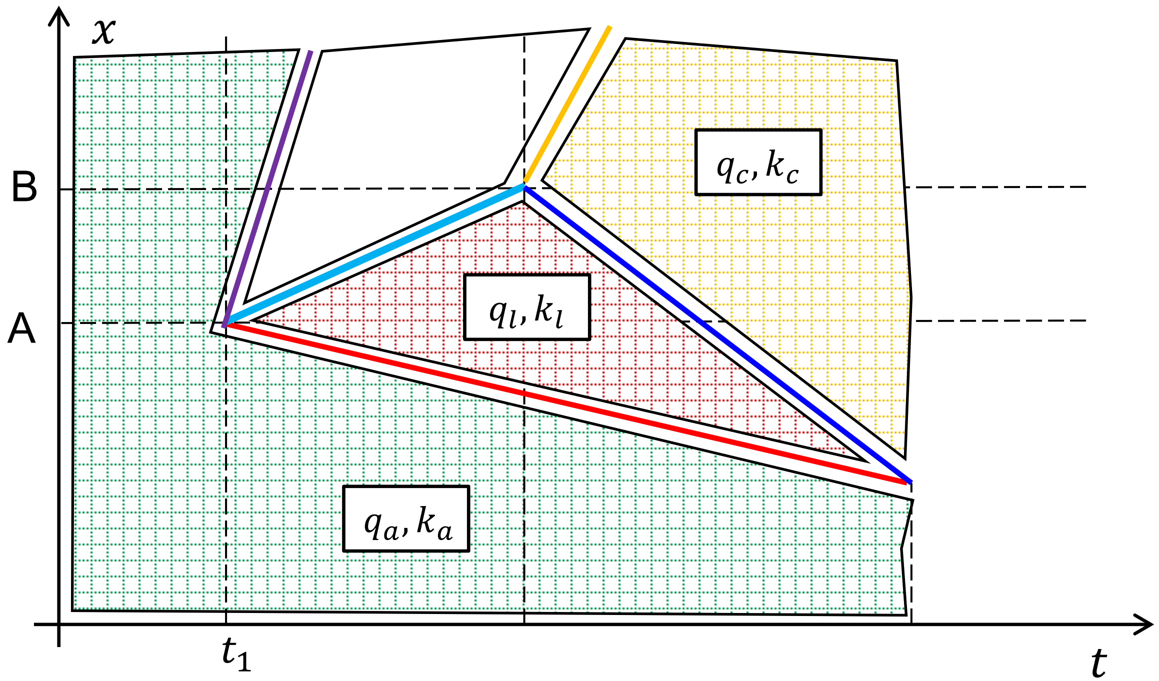

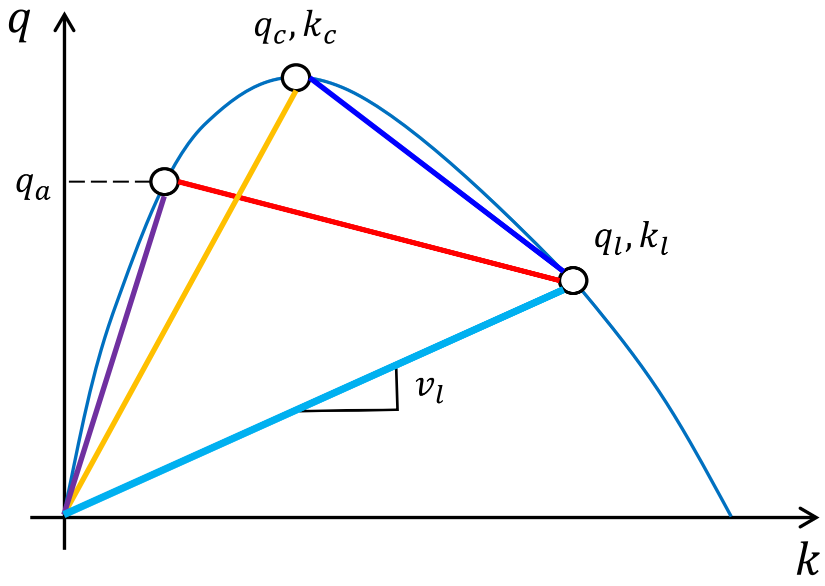

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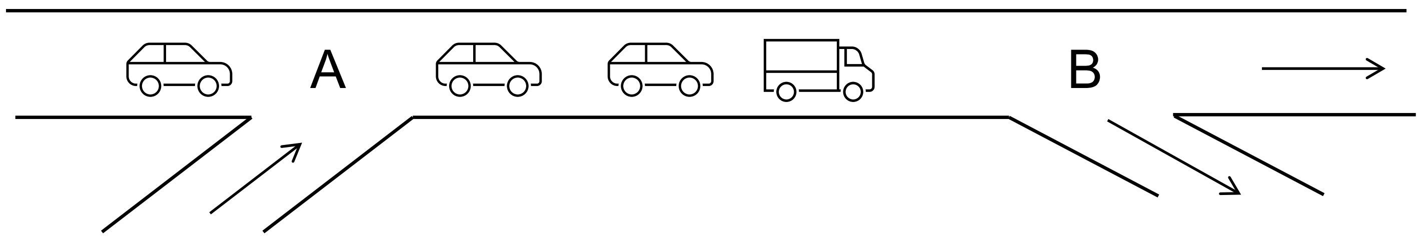

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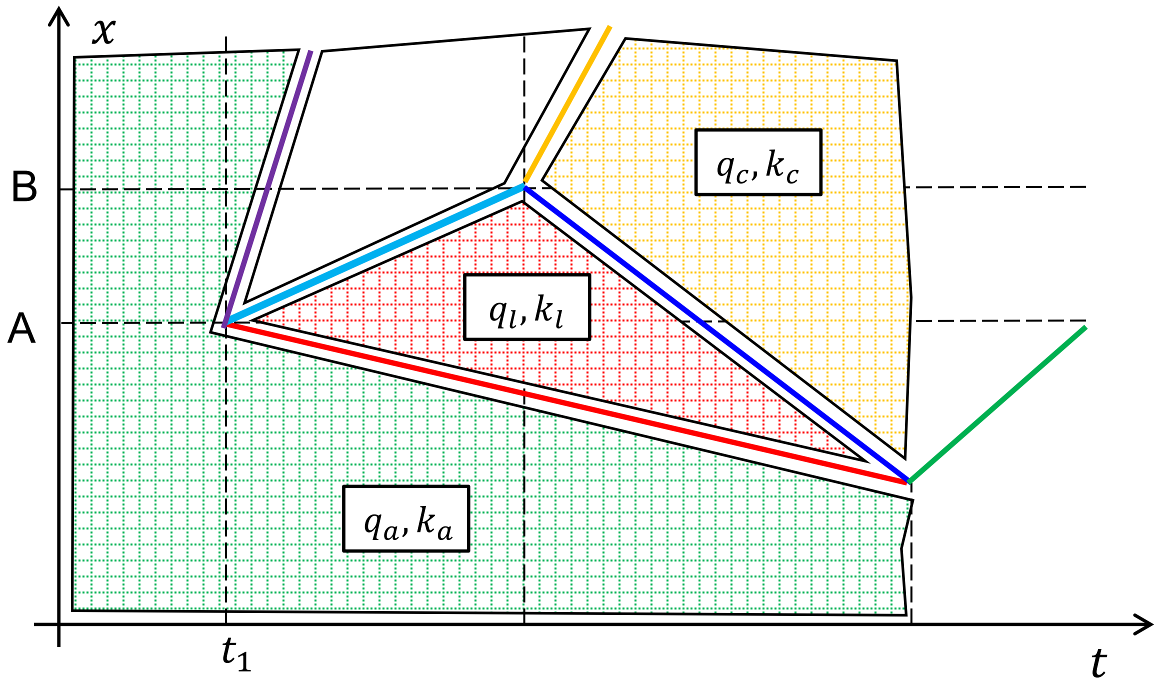

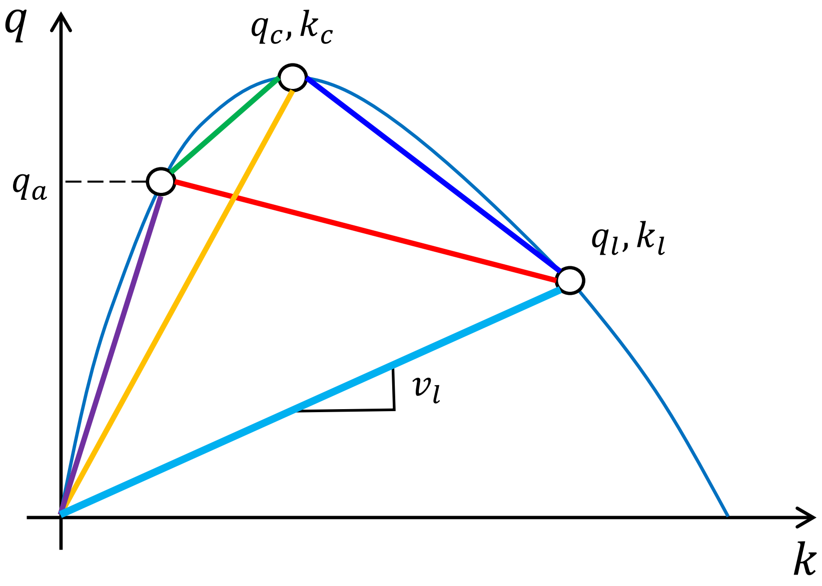

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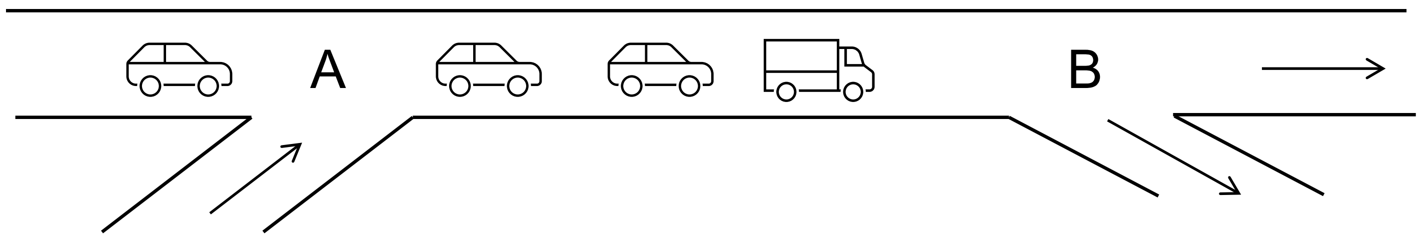

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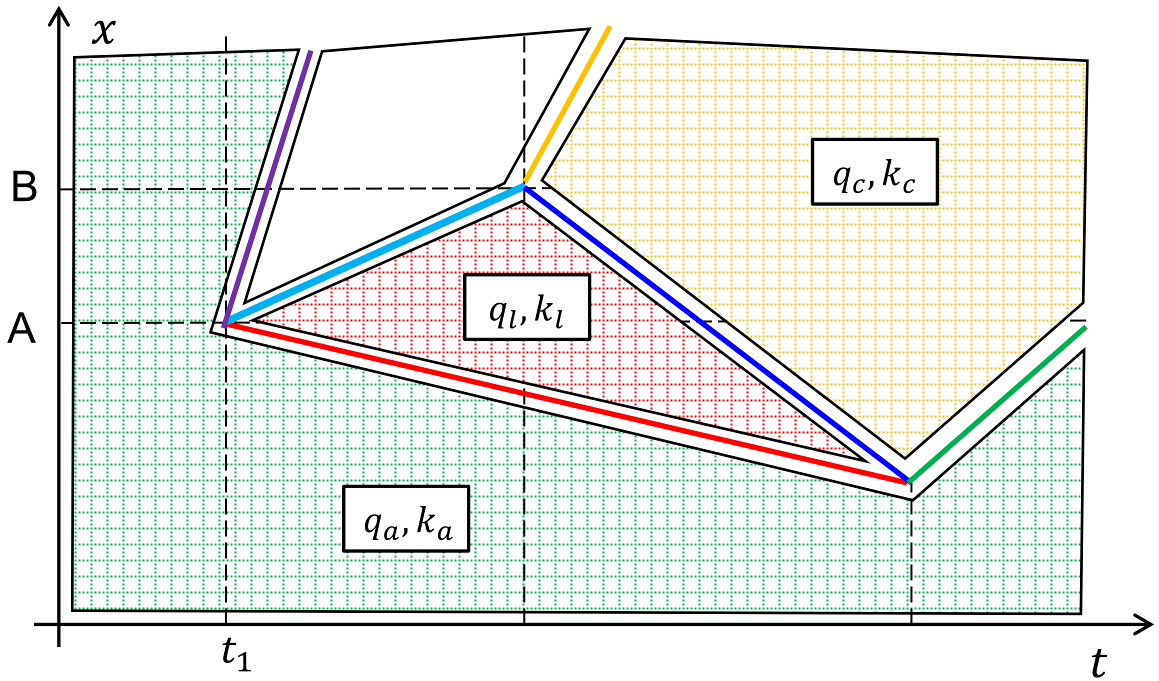

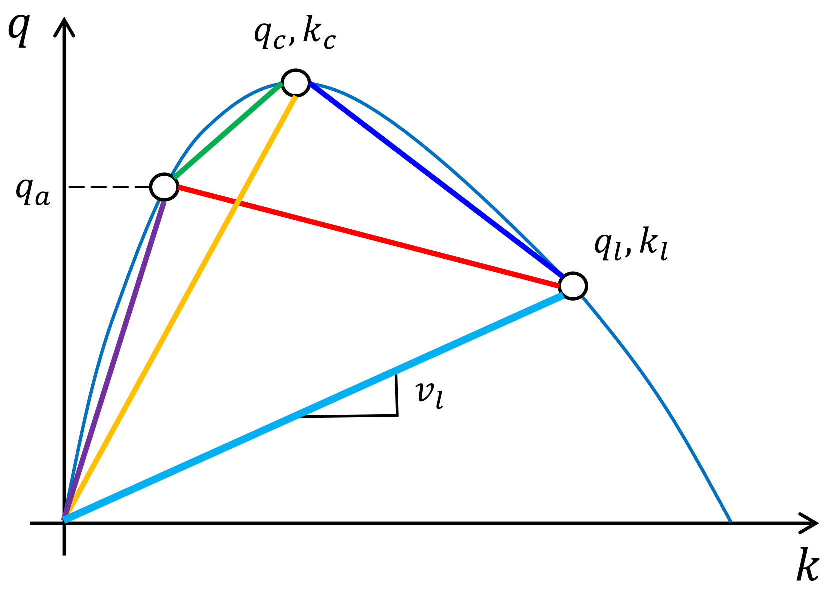

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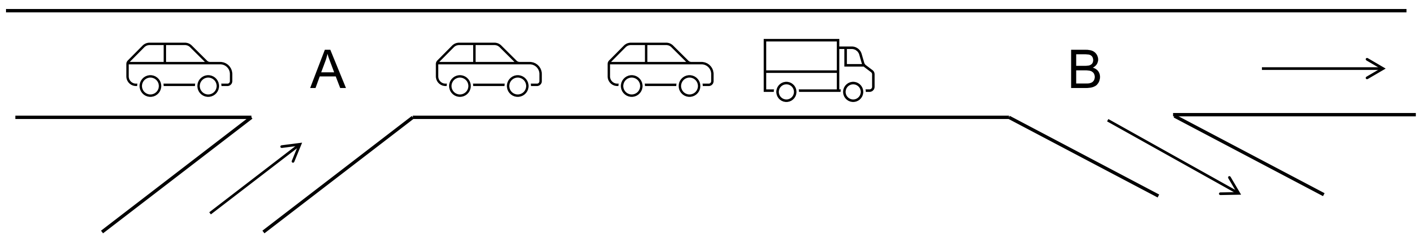
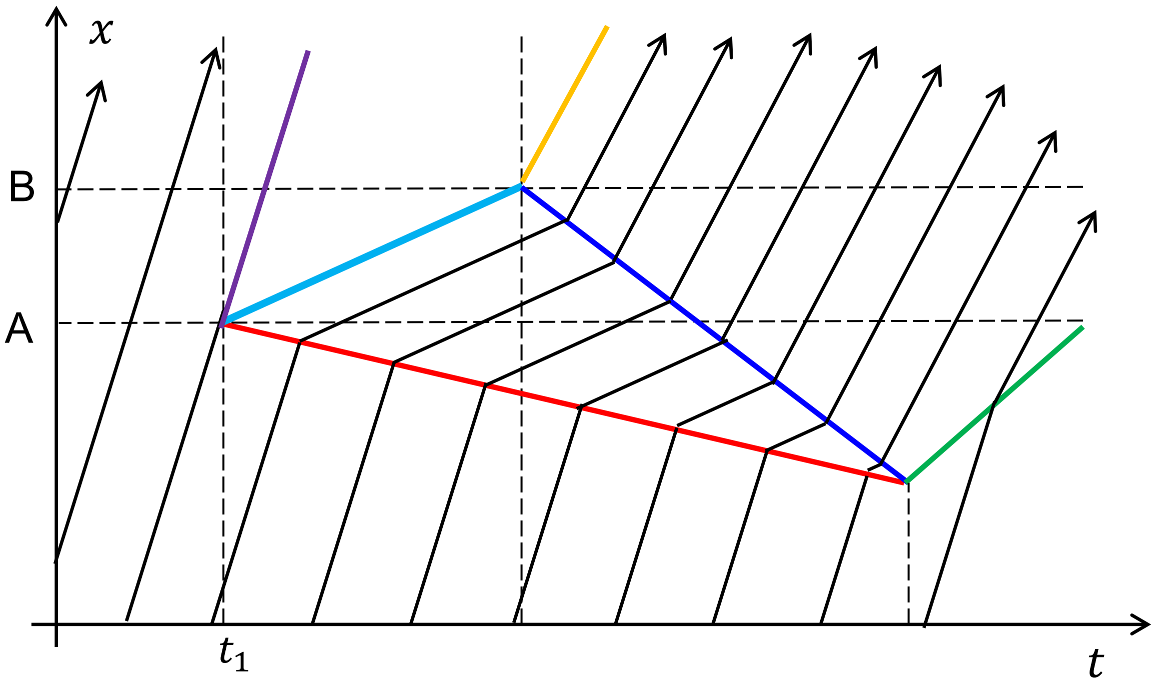
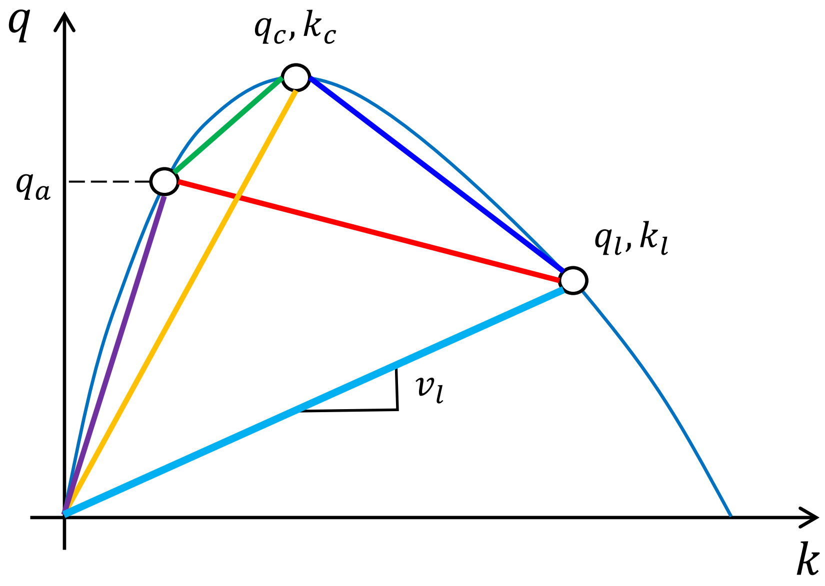


A truck enters the highway at t_1 with a slower speed v_l and exits at B. Traffic volume is q_a



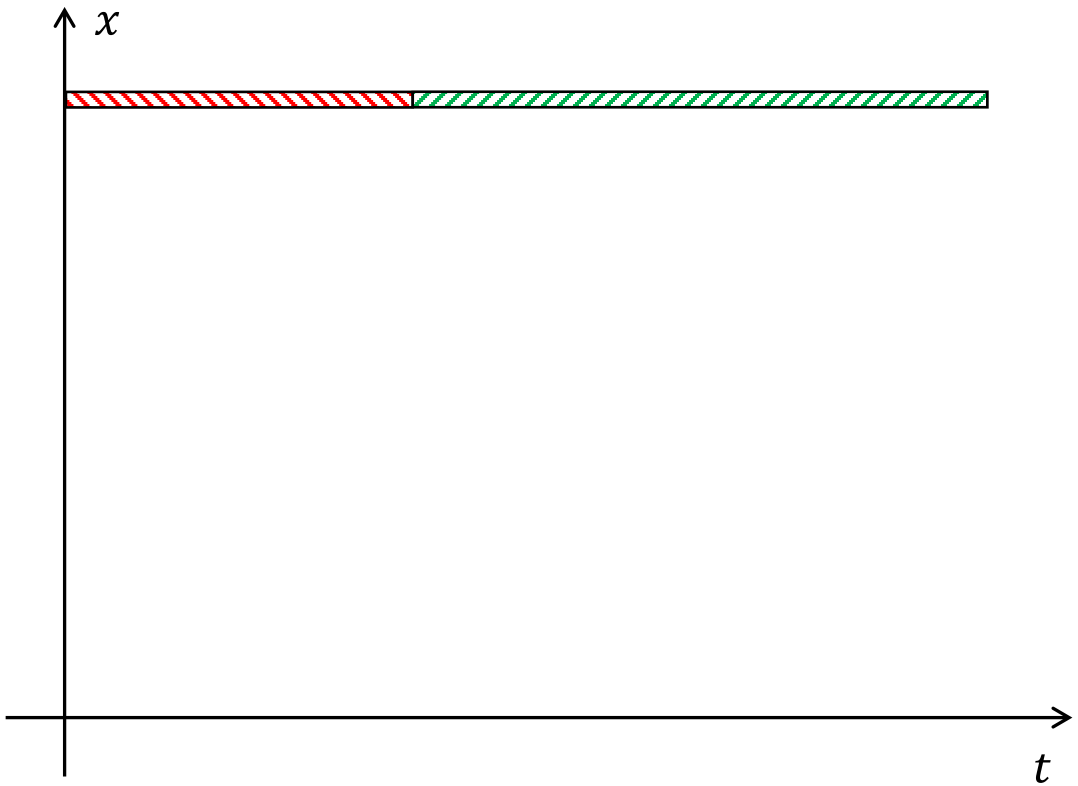


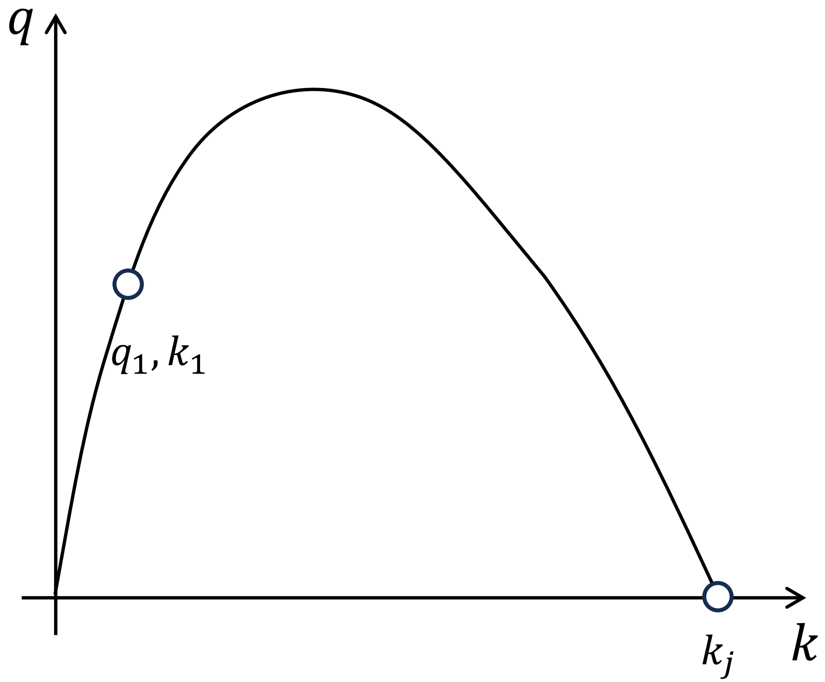
Outline

- ❑ A simplified shockwave solution
- ❑ Example 1: highway moving bottleneck
- ❑ Example 2: signalized intersection
- ❑ Calculating total delay with the TS diagram

Example 2: signalized intersection

- A roadway controlled by a traffic signal. The vehicle arrives at a constant arrival rate of q_1. The FD and traffic signal state are given. How to get the traffic state in a cycle





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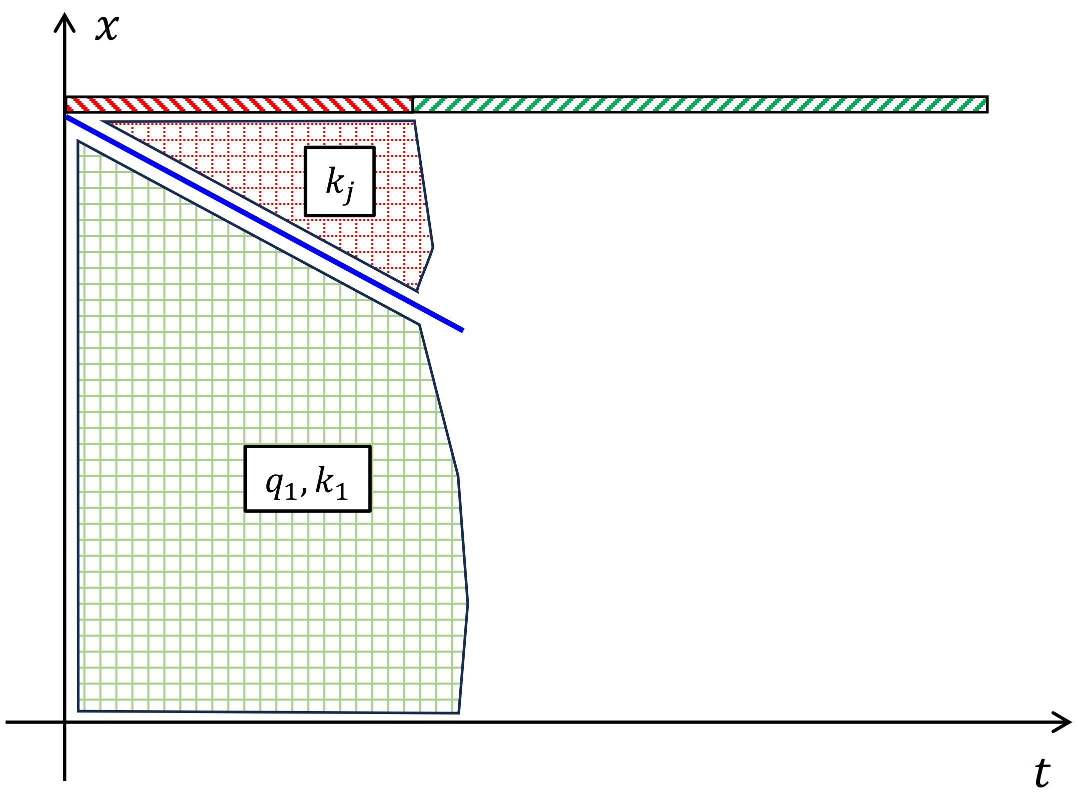

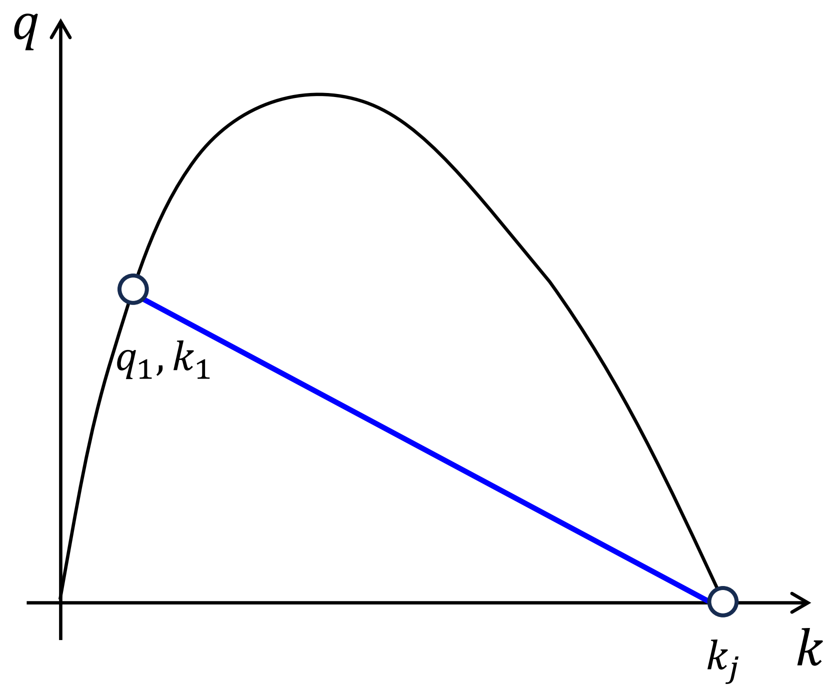

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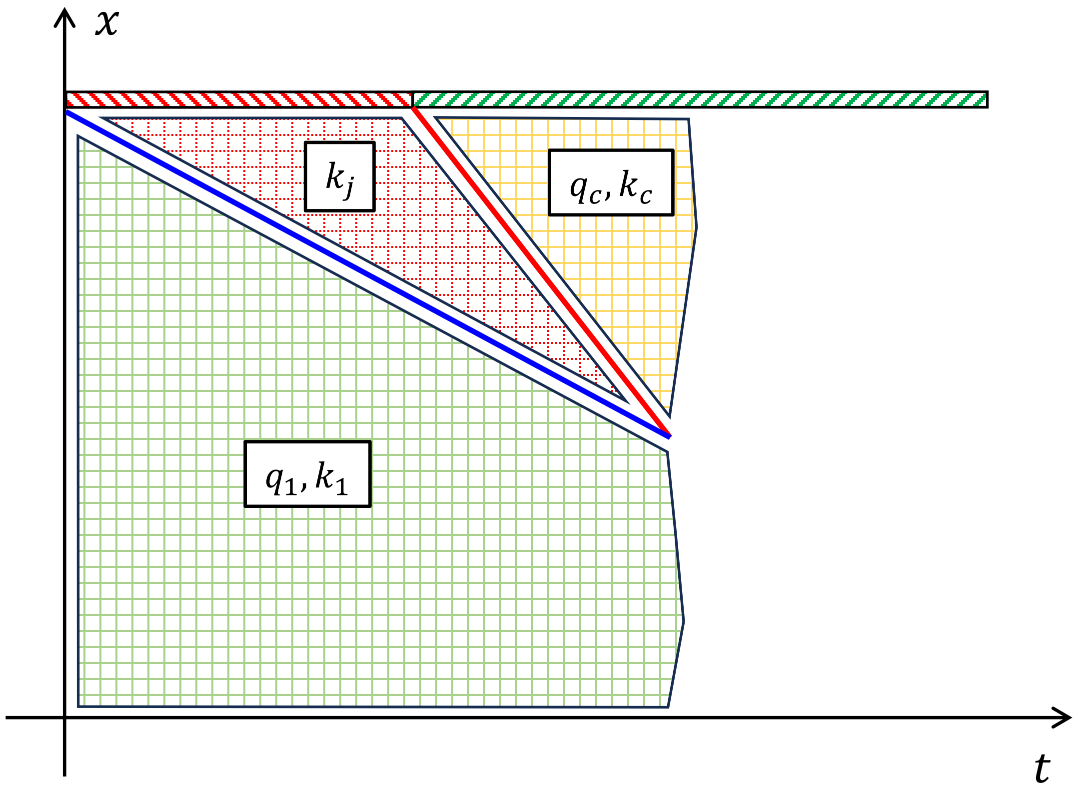

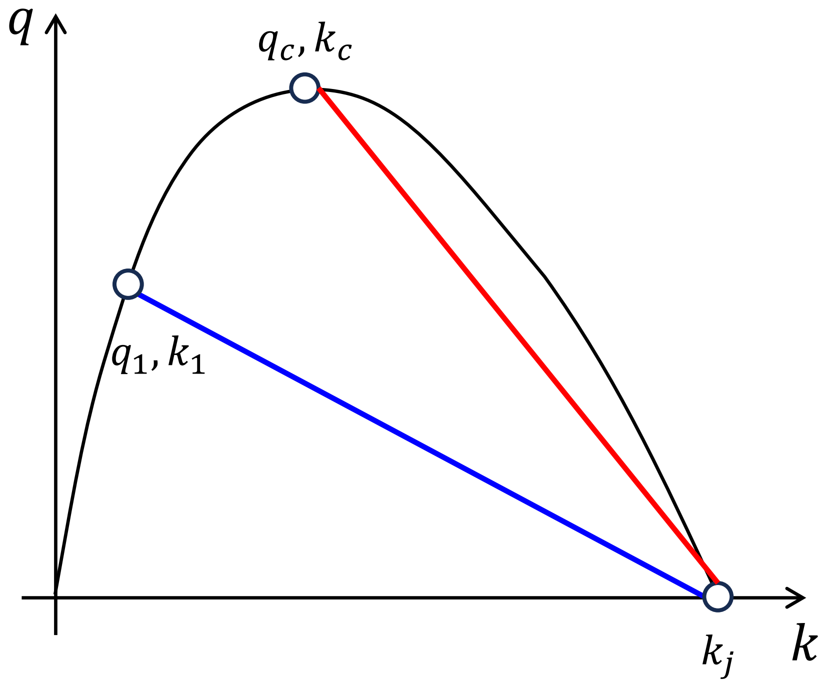

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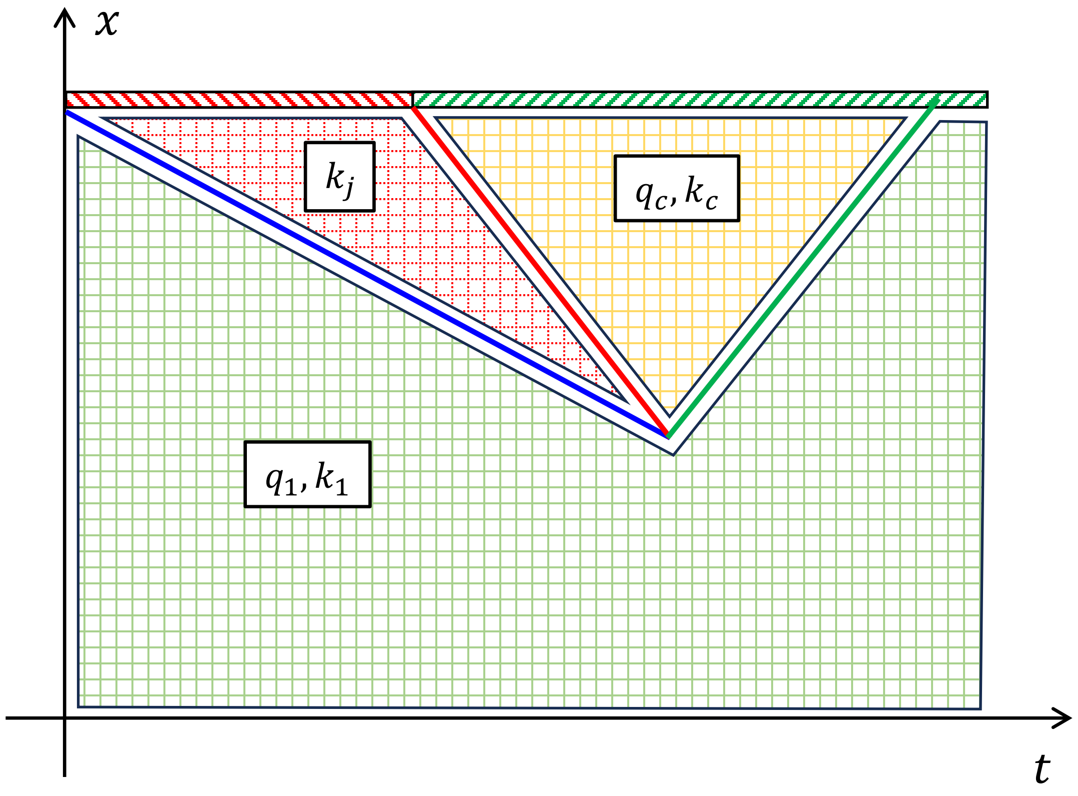
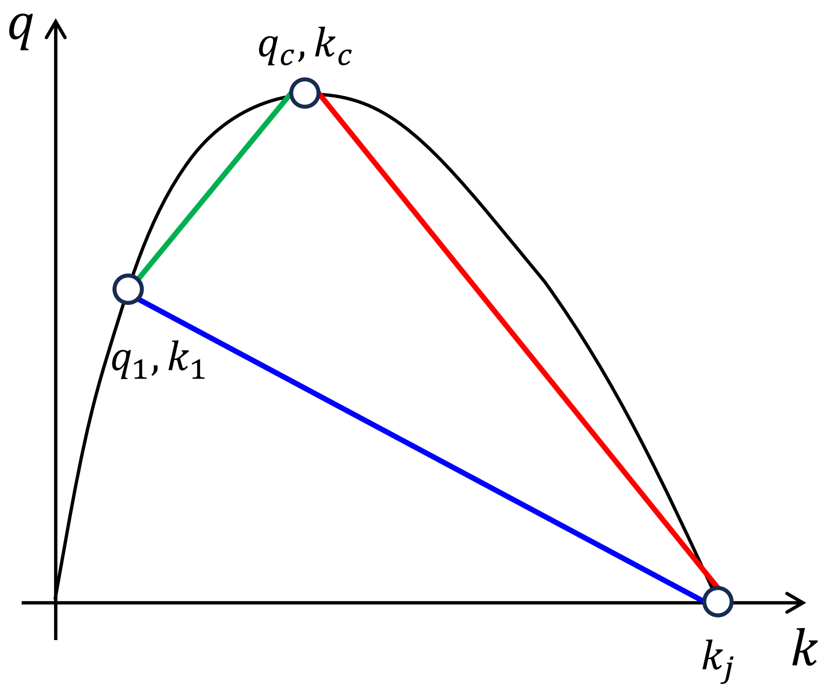
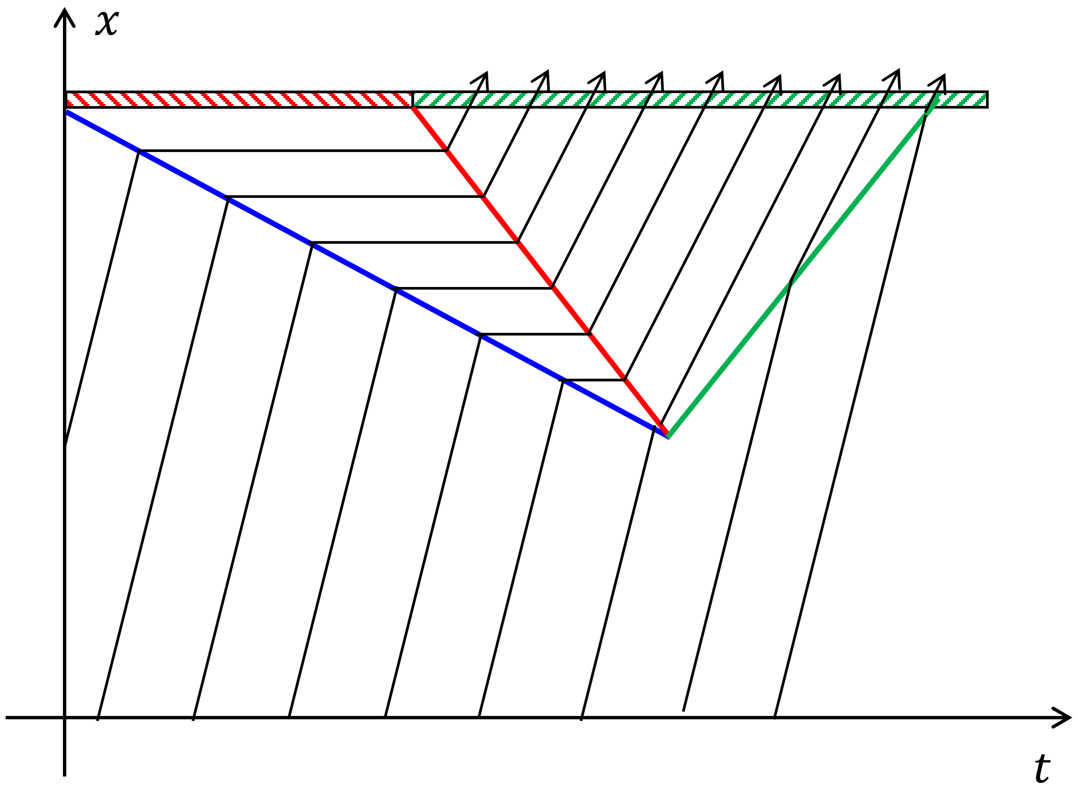
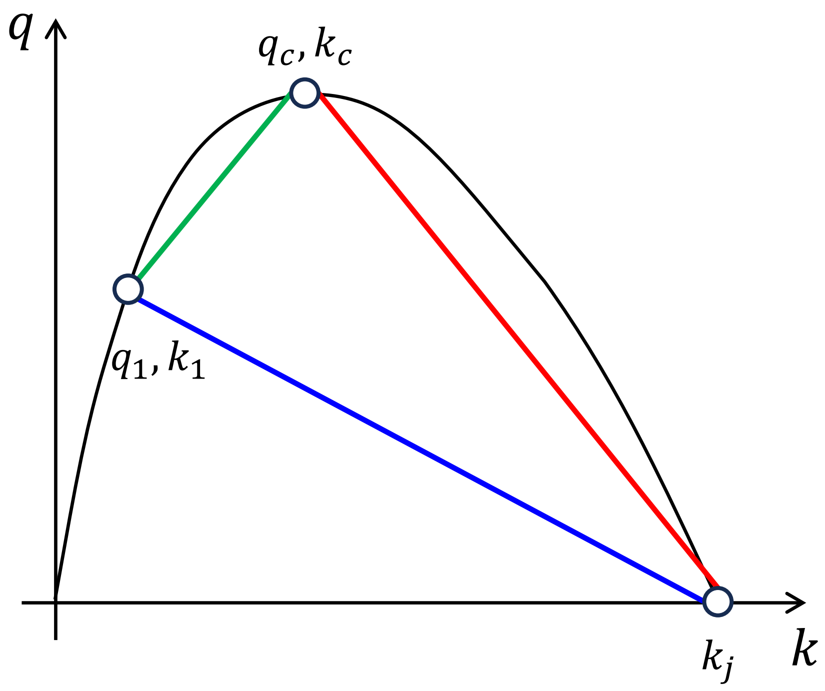




Example 2: signalized intersection

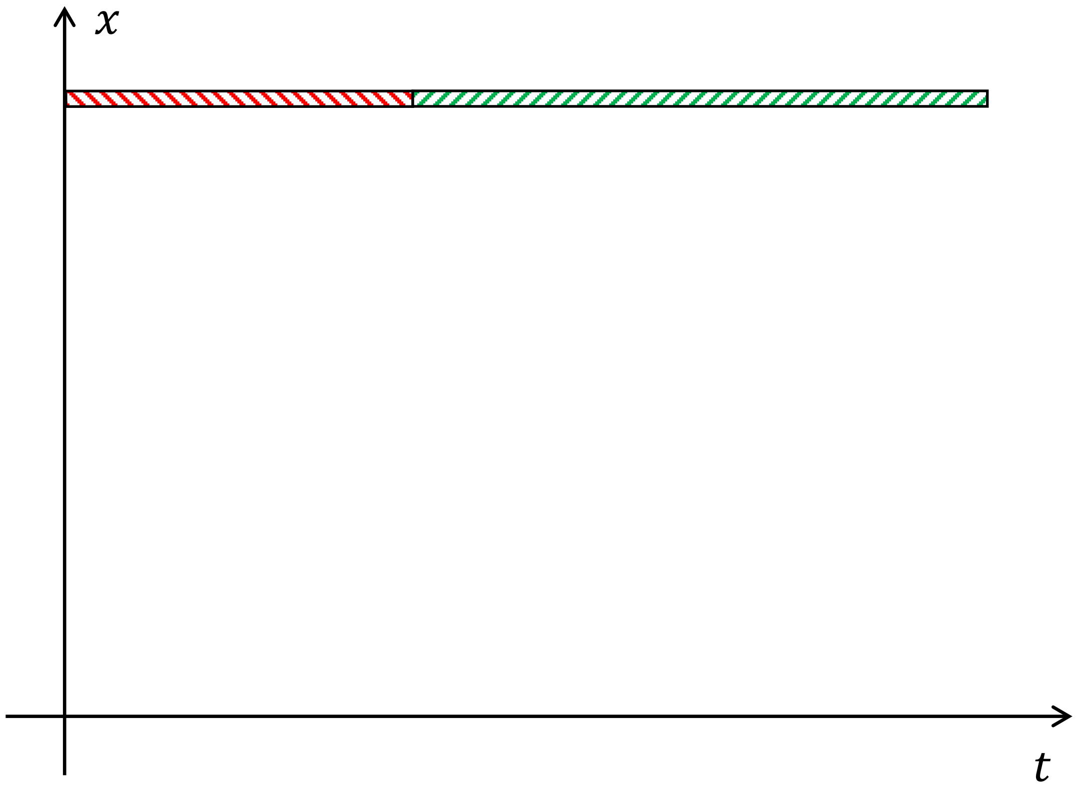
- Vehicle trajectories in the time-space diagram

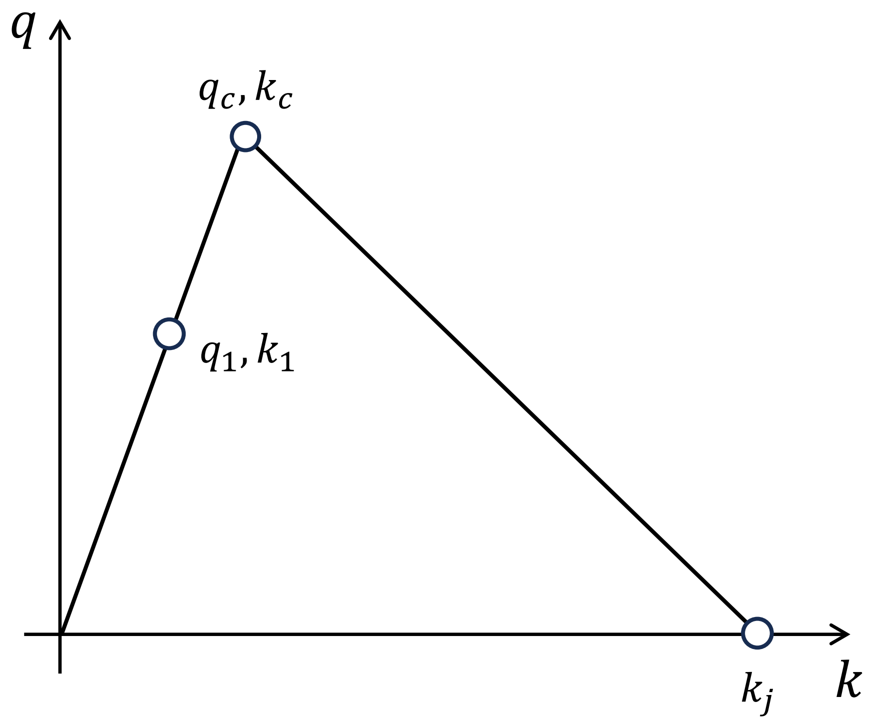




Example 2a: triangular fundamental diagram

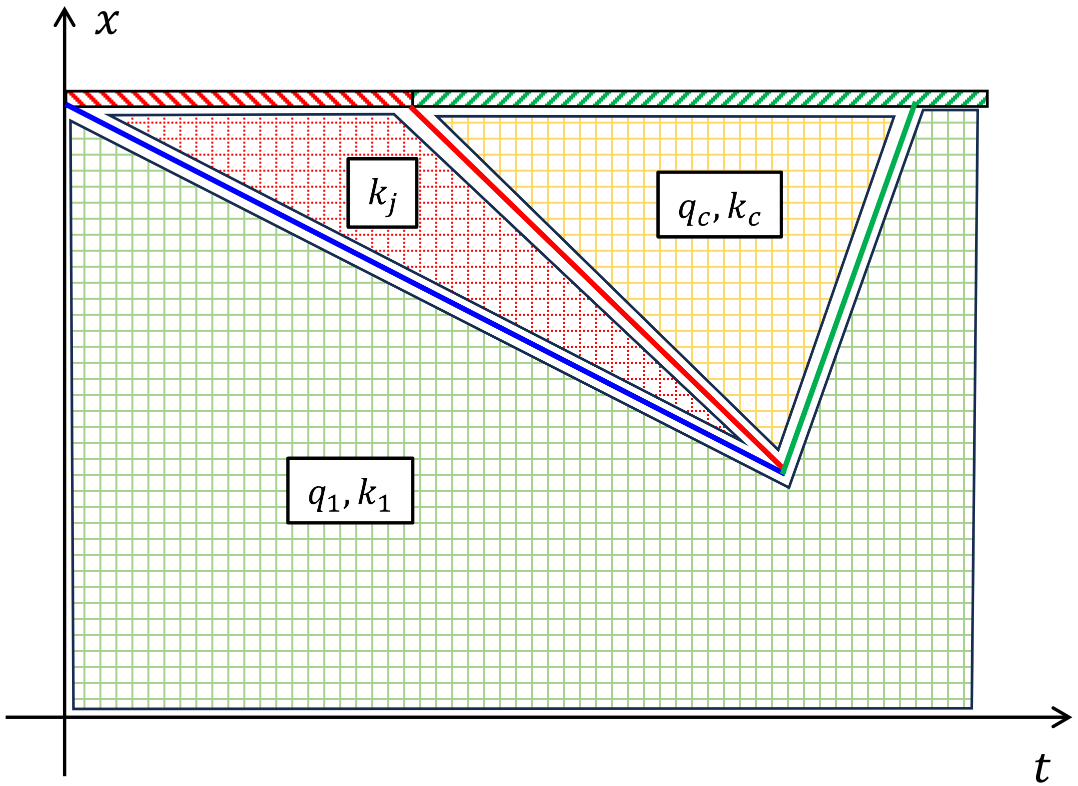
- What if we have a triangular fundamental diagram?
- What is the difference?

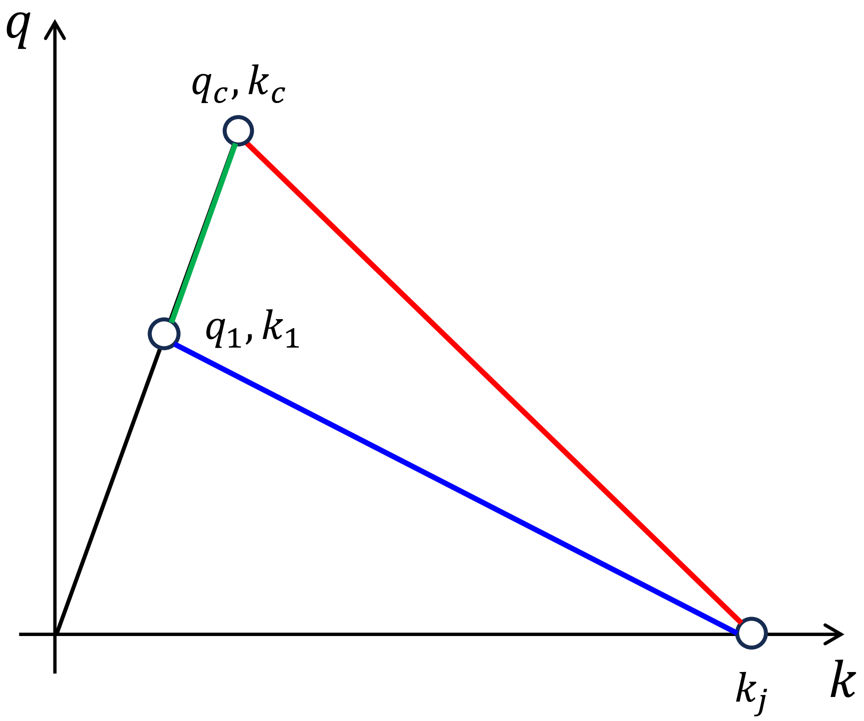




Example 2a: triangular fundamental diagram

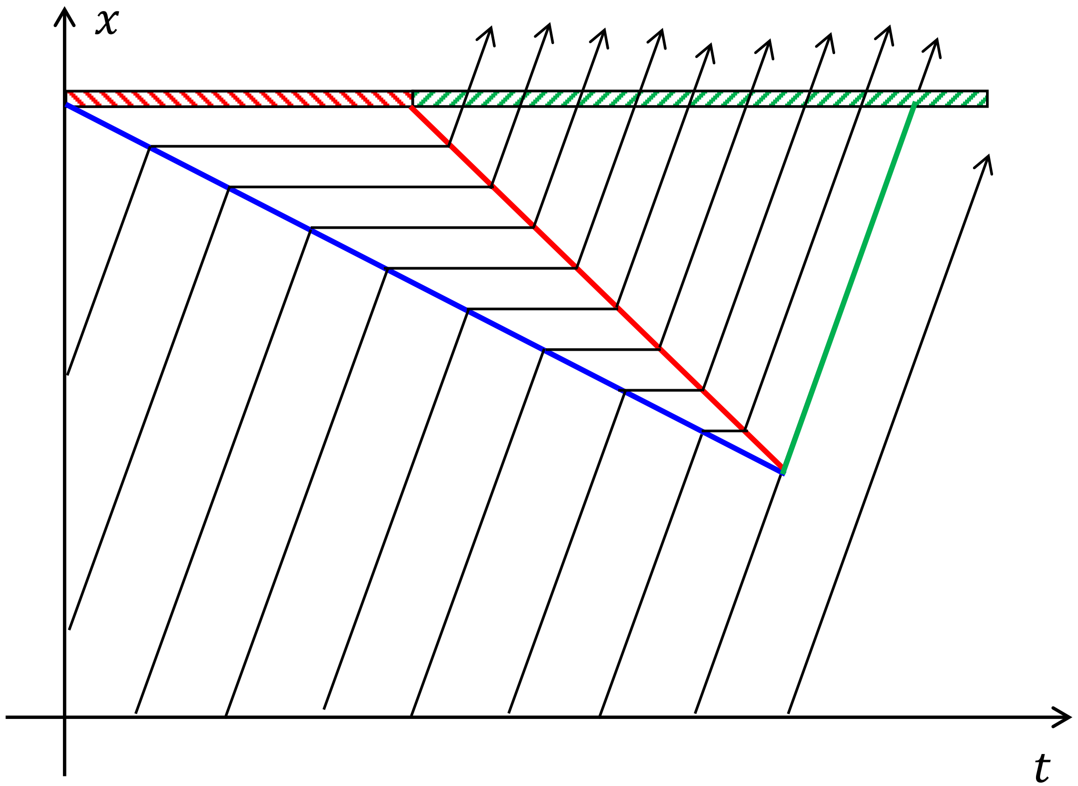
- Time-space diagram of signalized intersection under a triangular FD

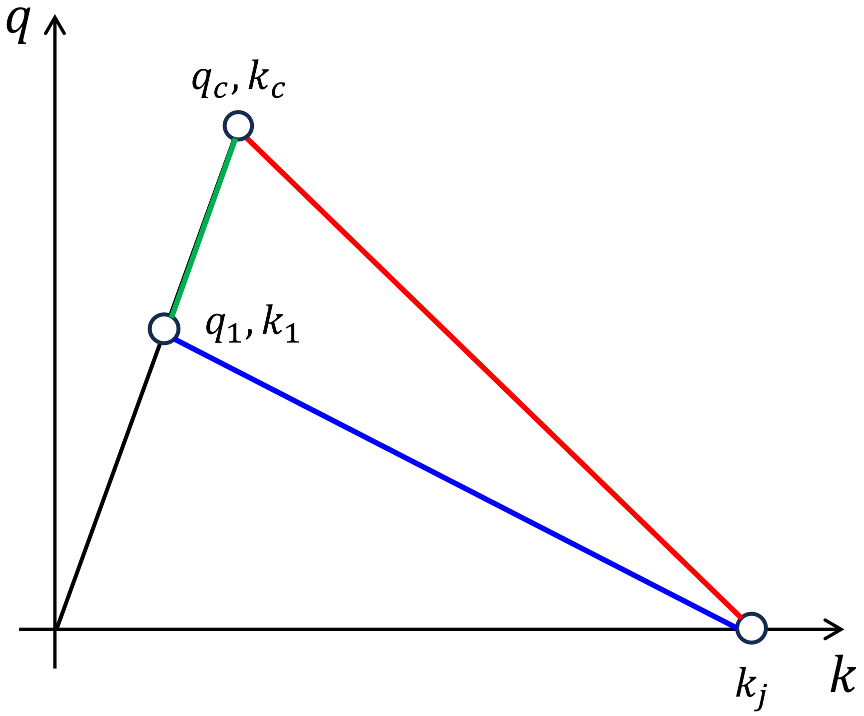




Example 2a: triangular fundamental diagram

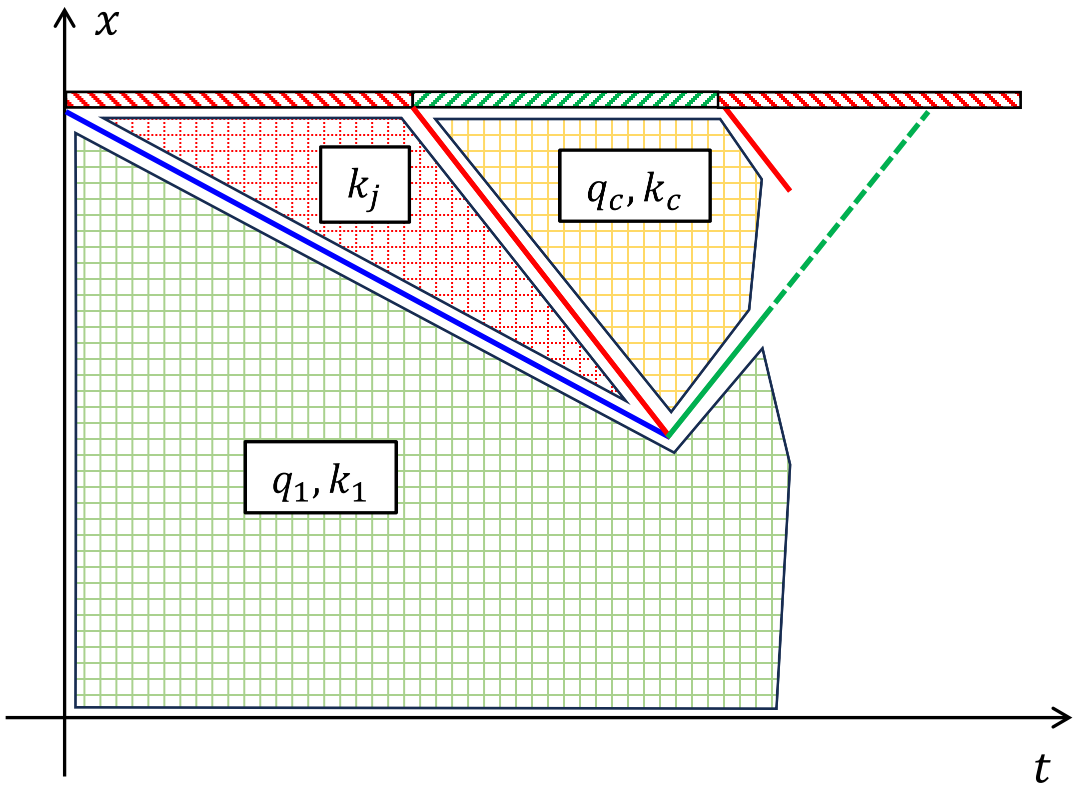
- Time-space diagram of signalized intersection under a triangular FD
  - Vehicle trajectories only have two states: stop and go (free-flow)

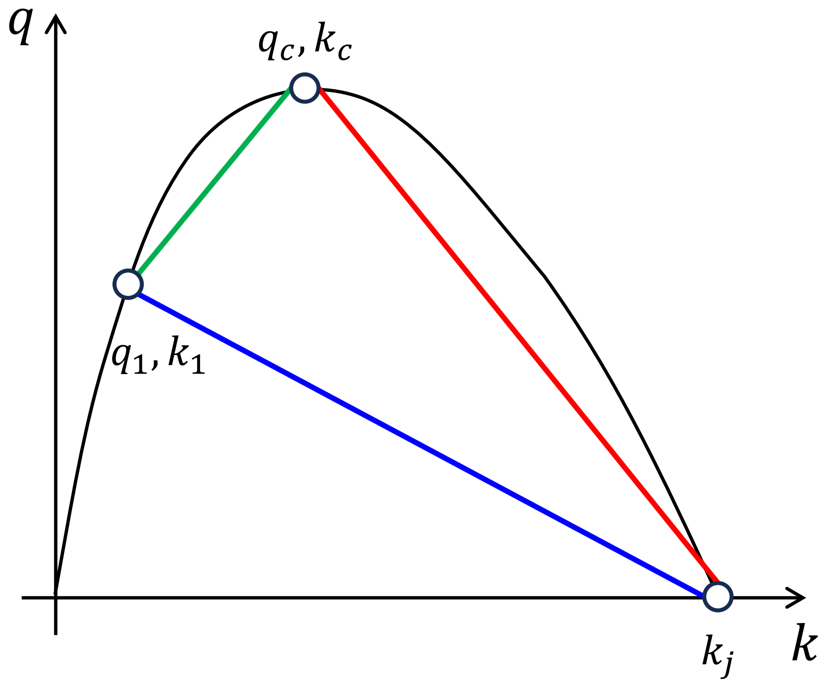




Example 2b: oversaturation

- If the green light duration is not sufficient, the queueing vehicle will not be discharged within a traffic cycle





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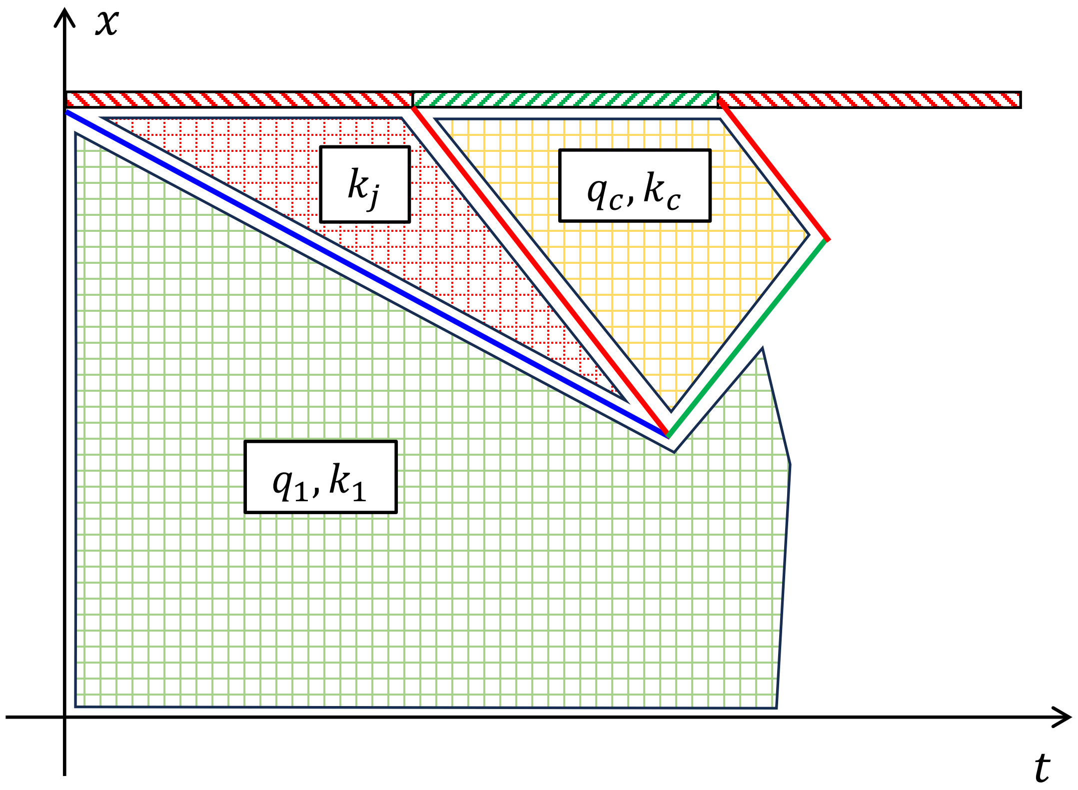

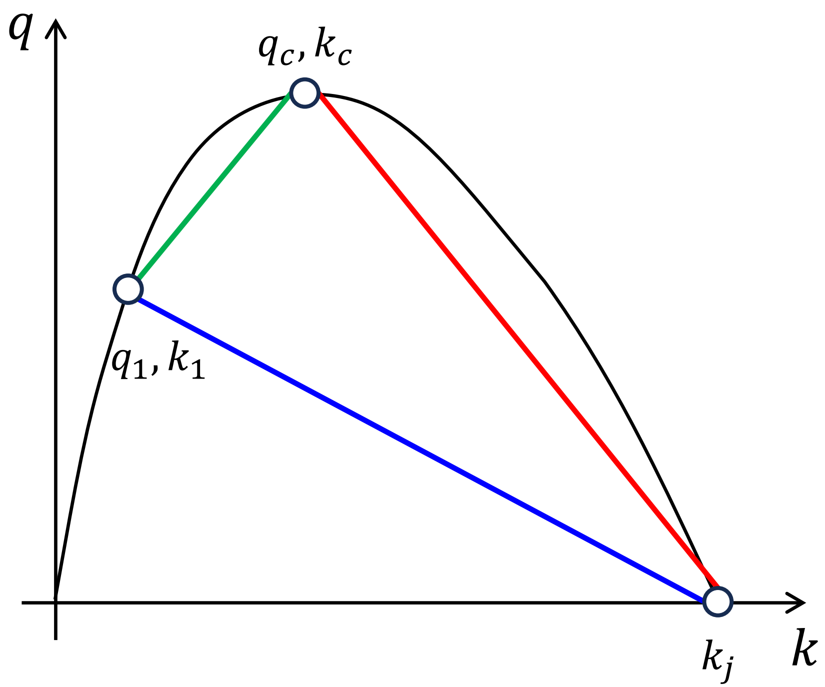

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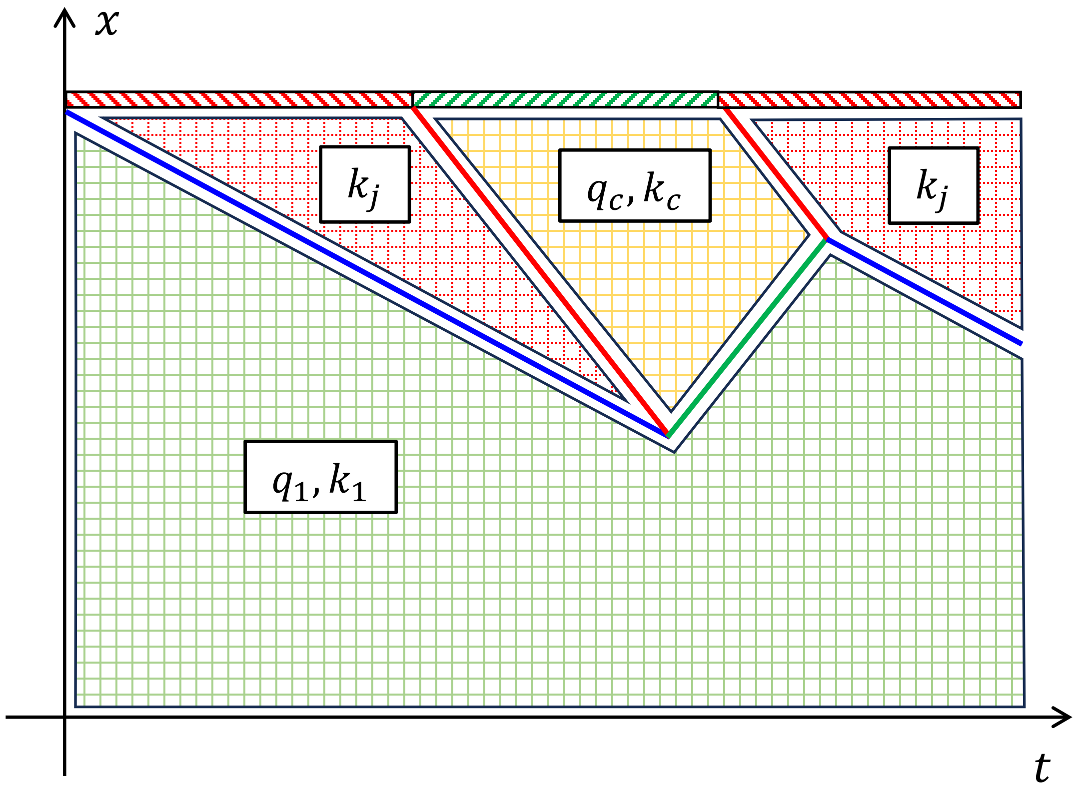
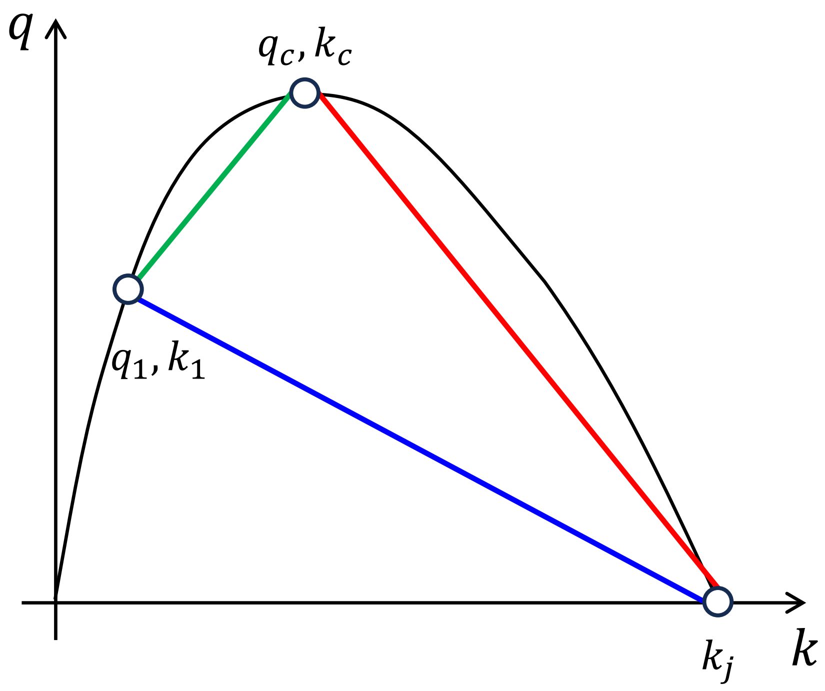
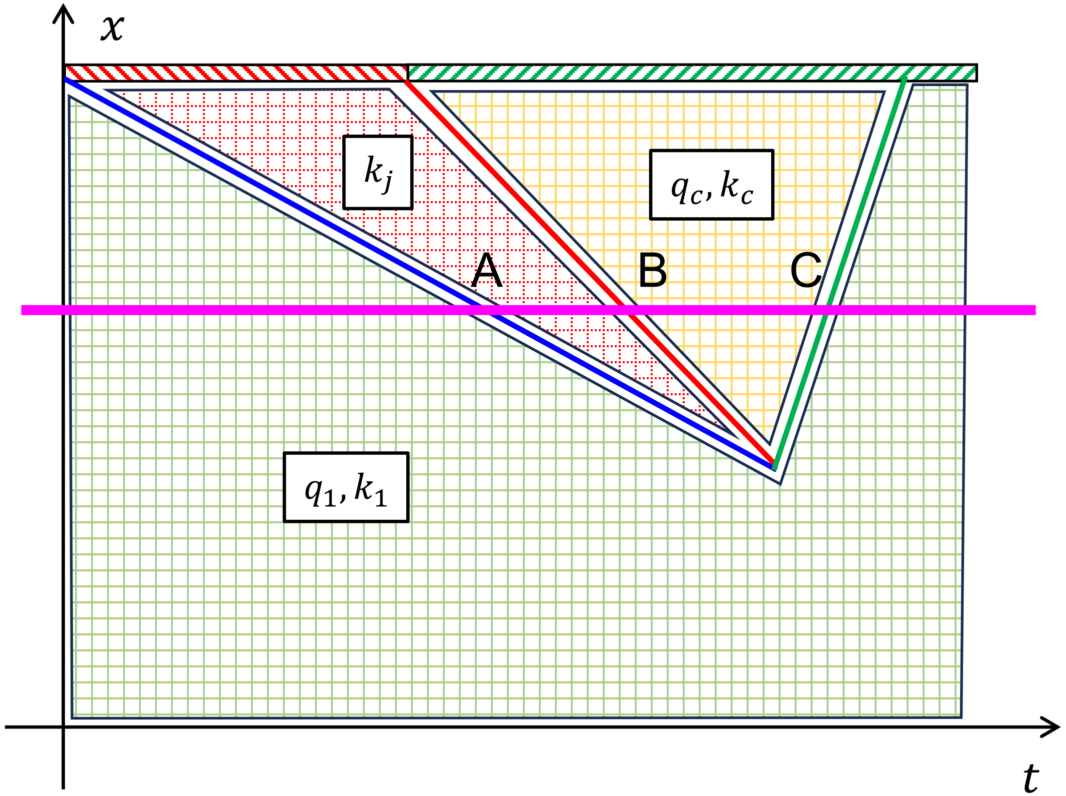
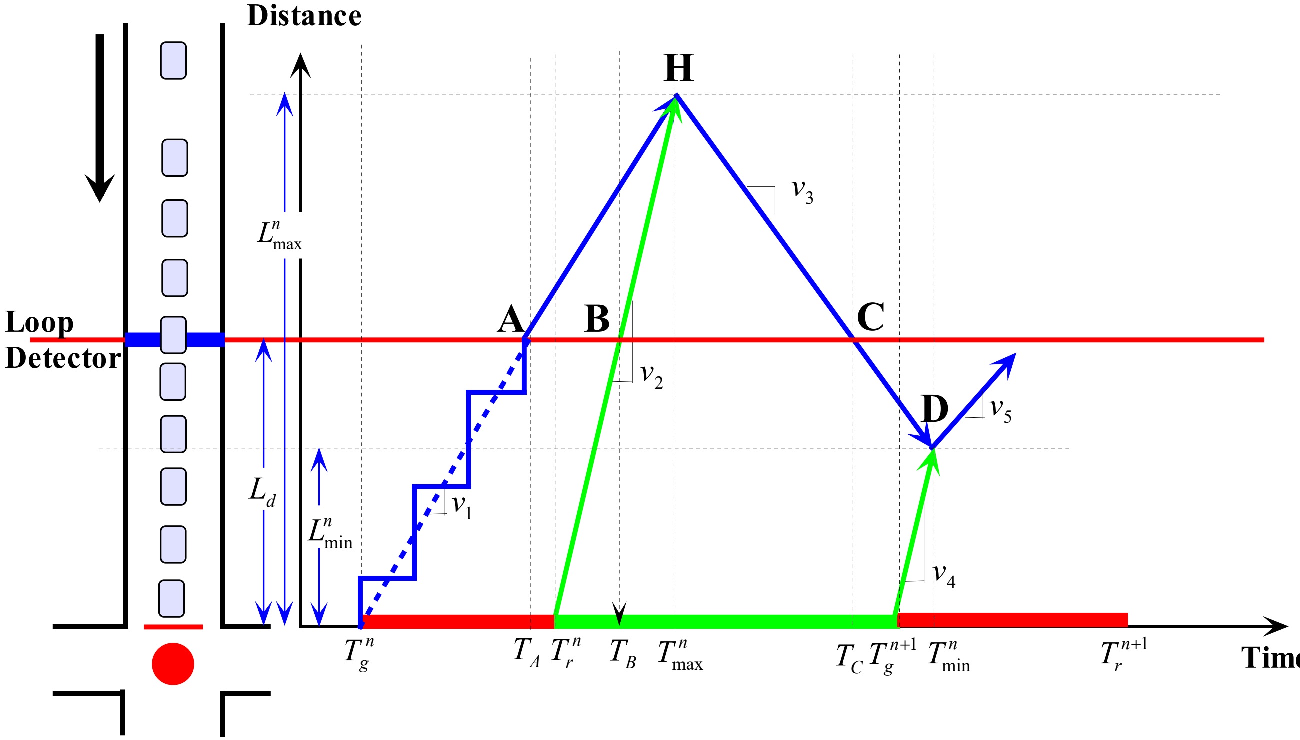




Queue length estimation with detector data

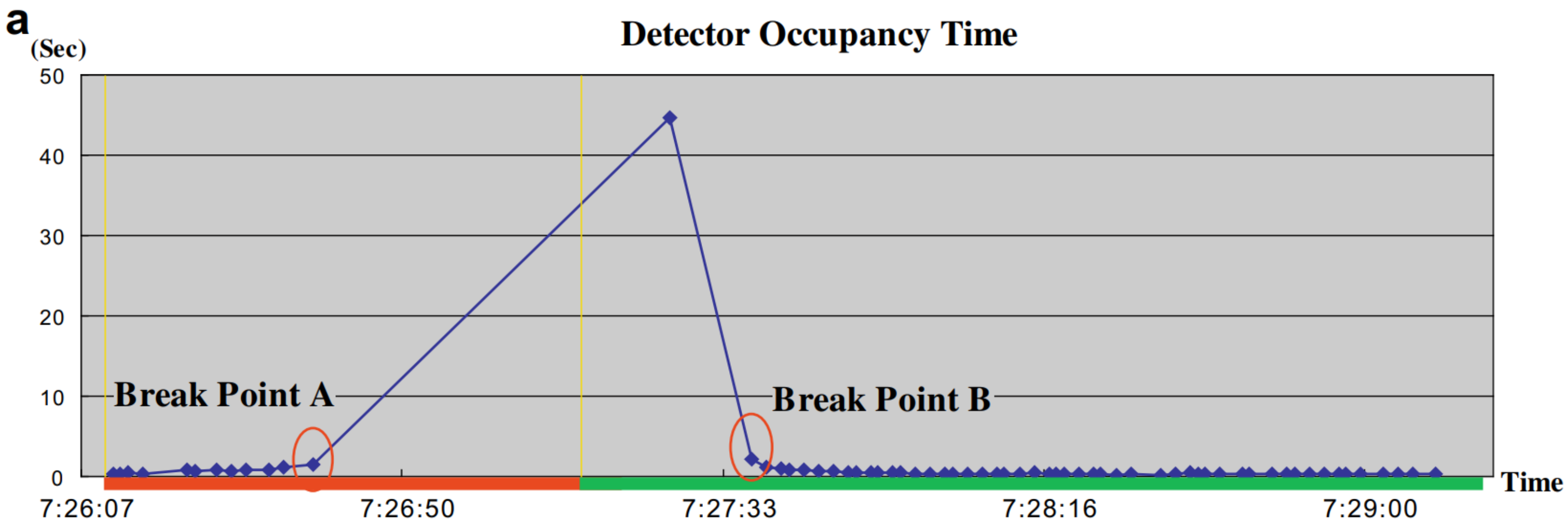
- We can detect the queue length given high-resolution detector data by identifying the boundaries (break points) between traffic states

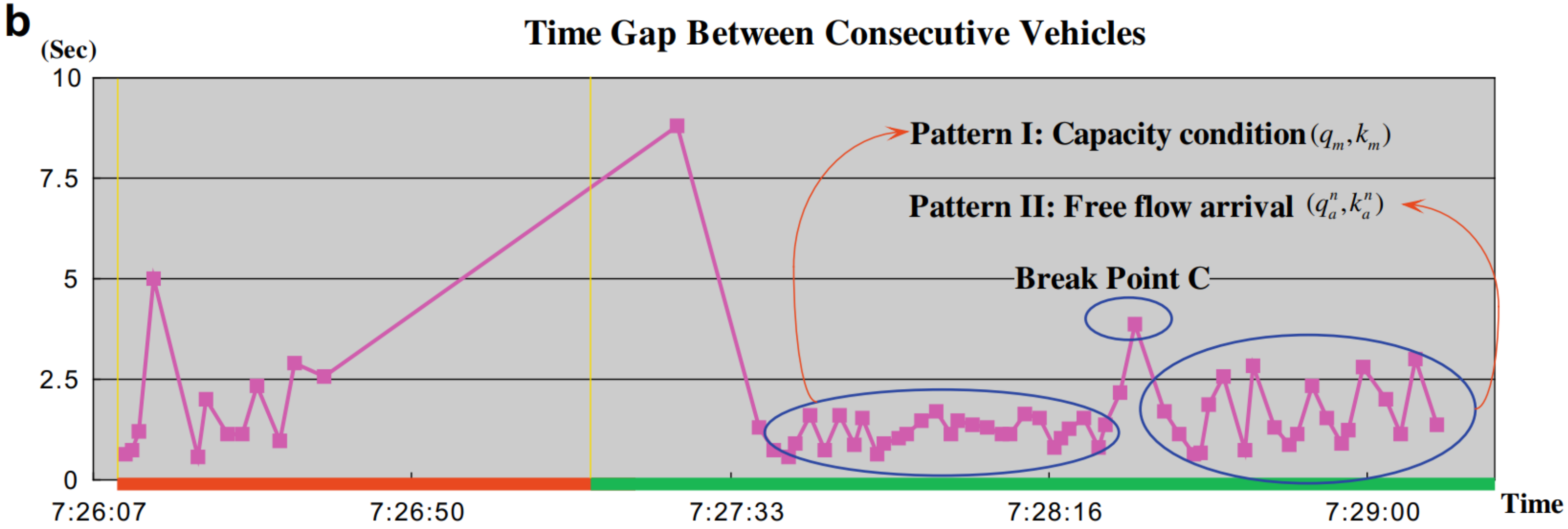


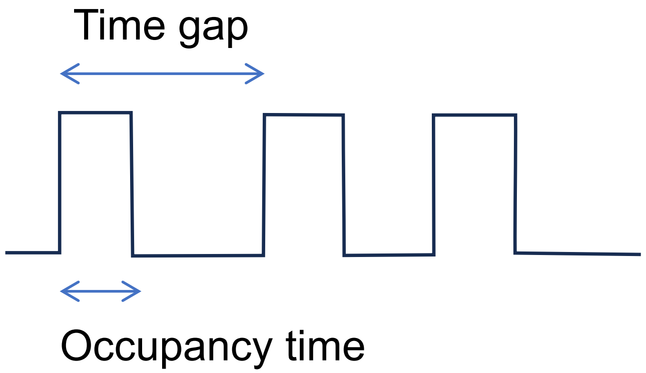


Liu, Henry X., et al. "Real-time queue length estimation for congested signalized intersections." *Transportation research part C: emerging technologies* 17.4 (2009): 412-427.

Detector data records







- Occupancy time: speed
- Time gap: headway (reciprocal of the traffic flow rate)

Outline

- ❑ A simplified shockwave solution
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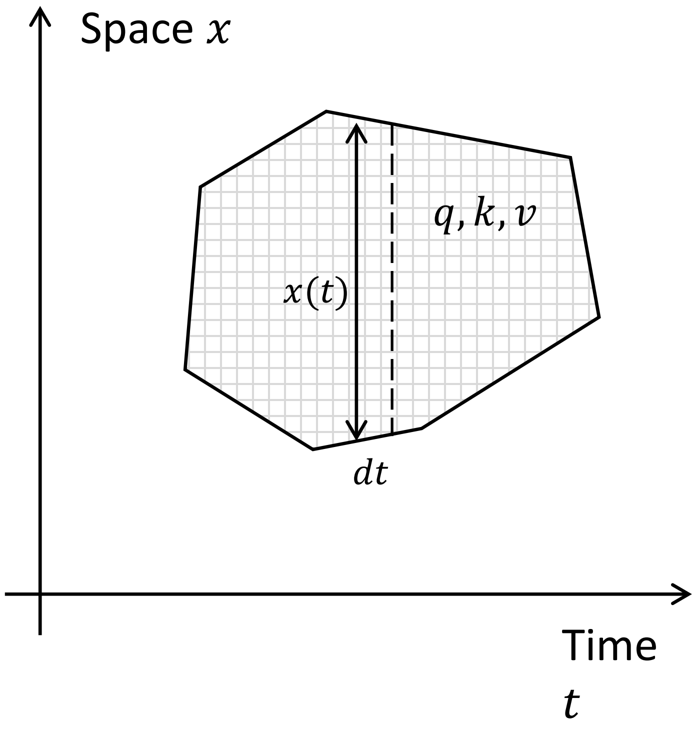
Total travel time in the TS diagram

- Total travel time equals to the density times the area in the TS diagram



- Number of vehicle in dashed area at time t

$$n(t) = x(t) \cdot k$$

- Total travel time from t to $t + dt$

$$n(t) \cdot dt$$

- Total travel time for vehicles in the shaded region:

$$TTT = \int n(t)dt = k \int x(t)dt = k \cdot S_0$$

(S_0 is the area of shaded region in the TS diagram)

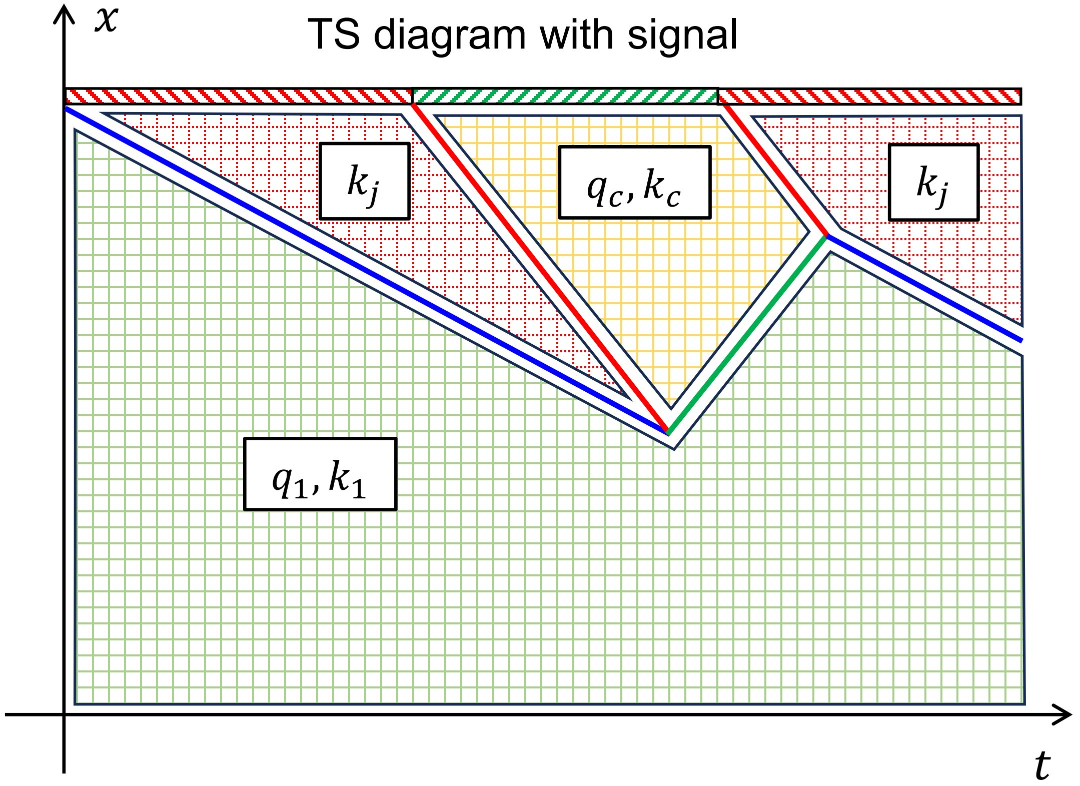
- Similarly, we have the total travel distance:

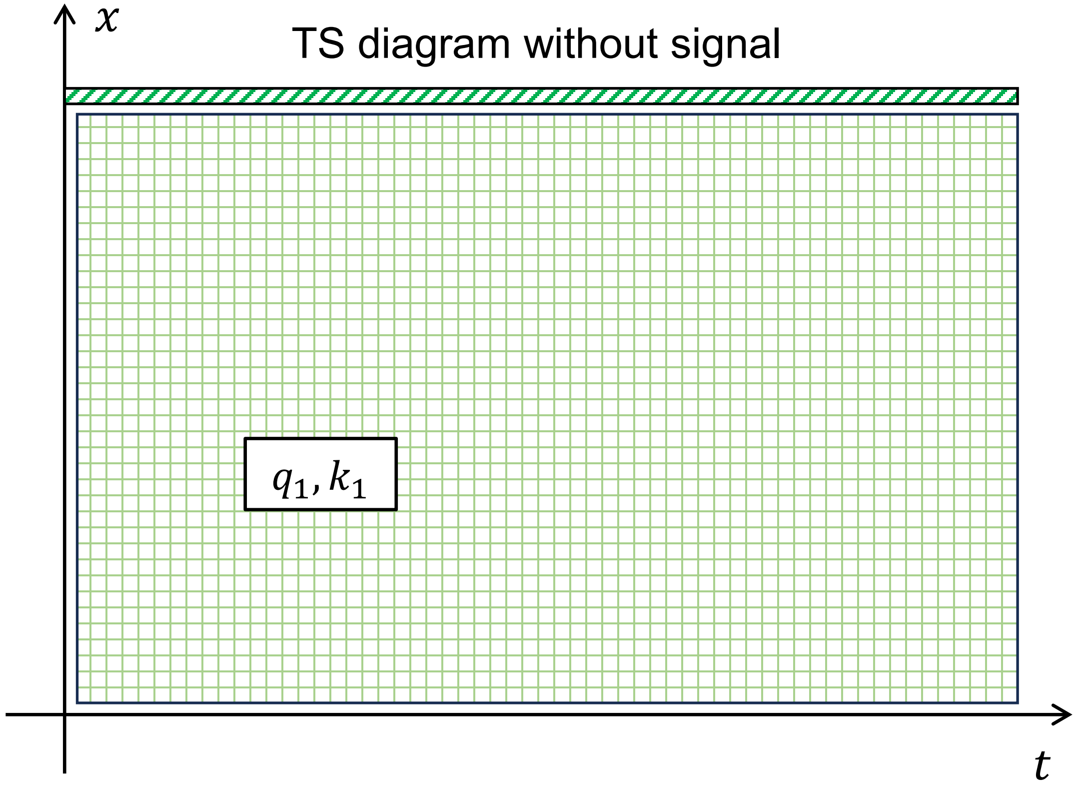
$$TTD = q \cdot S_0$$

Check units!

Calculating delay caused by traffic signal

- Based on the total travel time equation, we can calculate the total delay caused by traffic signals





$$Total\ delay = TTT_w - TTT_{w/o} = \sum_i k_i S_i - k_1 \sum_i S_i$$

(S_i denotes the area of each region in the TS diagram)

Level of service analysis

Traffic signal optimization

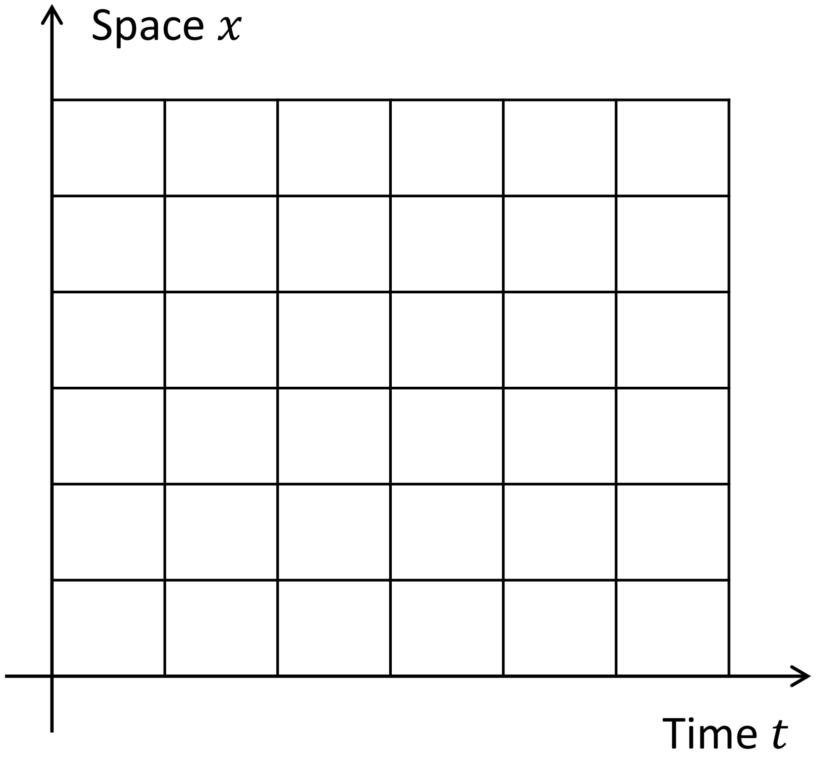
- With the delay evaluation, we can formulate the traffic signal optimization program:

$$\min_{g_i} \sum_i D_i(C_i, g_i, q_i)$$

$$s.t. \sum g_i = C$$

- This is just a simple demonstration, there are many details regarding traffic signals which will be covered in the second part of this course

Edie's definition of average flow and density

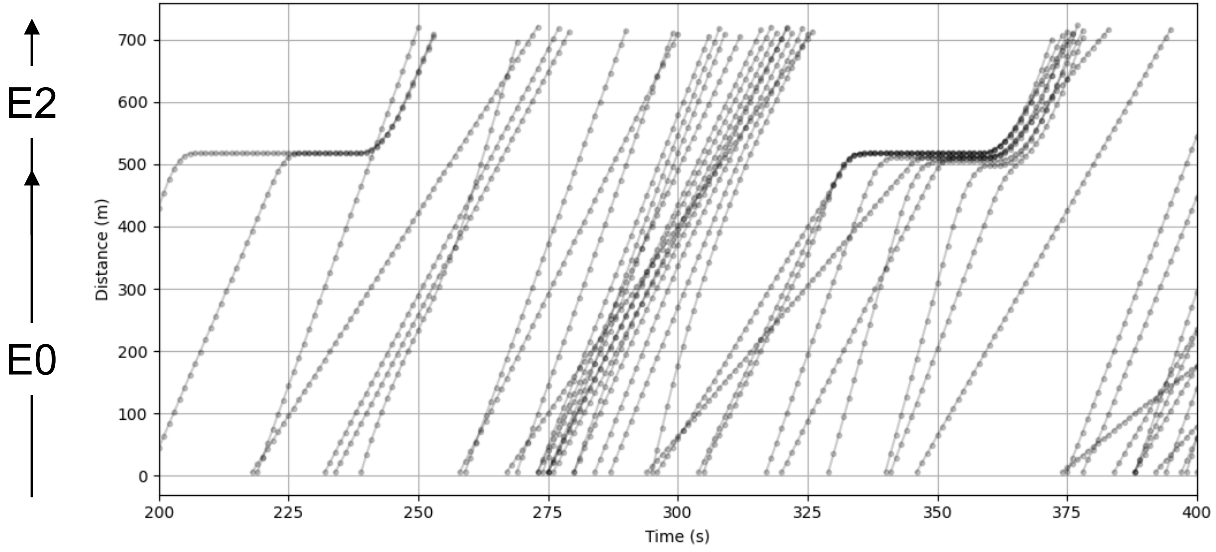


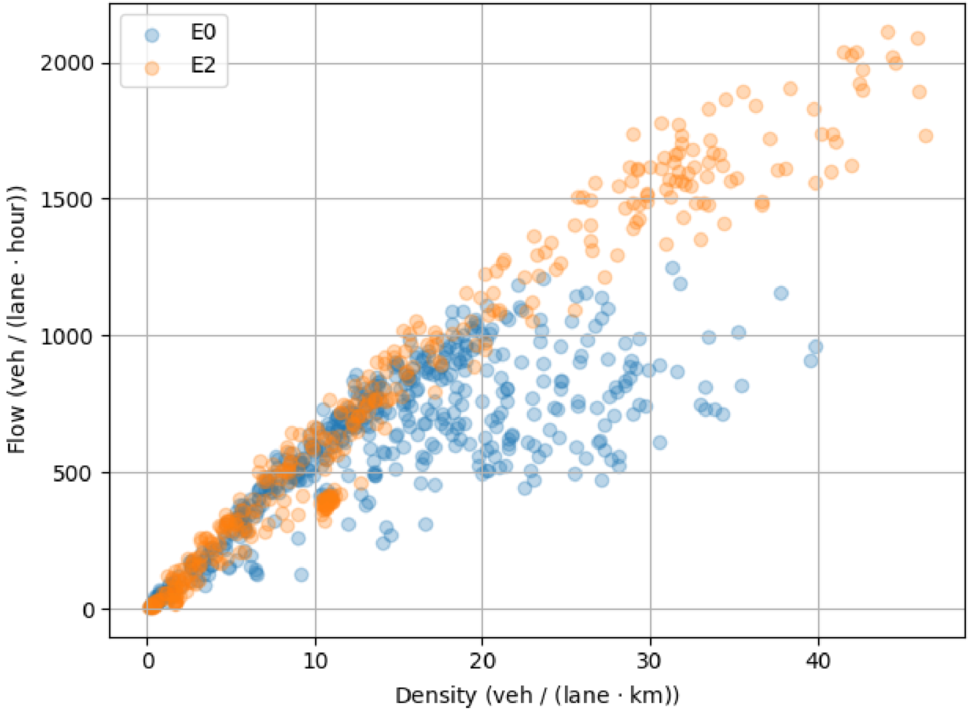
$$\begin{cases} TTT = \int n(t)dt = k\int x(t)dt = k \cdot S_0 \\ TTD = q \cdot S_0 \end{cases}$$

$$\Rightarrow \begin{cases} \bar{k} = \dfrac{TTT}{S_0} = \dfrac{TTT}{\Delta x \Delta t} \\ \bar{q} = \dfrac{TTD}{S_0} = \dfrac{TTD}{\Delta x \Delta t} \end{cases}$$

Edie, L. C. (1963). Discussion of Traffic Stream Measurements and Definitions.

Empirical FD from vehicle trajectories





Tips for generating empirical FD

(Key: each dot in FD should represent a relatively uniform traffic state)

- Careful selection of the spatial and temporal resolutions
- Set various inputs and bottlenecks such as scatters can cover all conditions

Homework assignment

- Homework 1
- Due time: 09/23
- (Homework submitted after the deadline without a valid reason will be accepted with a maximum possible score of 80%)